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AND

DISCUSSIONS

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(A constant effort is made to supply technical material to Society members, over the entire range of possible interest. In so far as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement.—Ed.)

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

HIGHWAY BRIDGE FLOORS

A SYMPOSIUM

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 1, 1948.**

LABORATORY RESEARCH ON CONCRETE BRIDGE FLOORS

By F. E. Richart,[1] M. ASCE

1. Purpose and Scope

A program of systematic research, which will be designated the "Concrete Slab Investigation" in this Symposium, was begun in September, 1936, as a cooperative project between the Engineering Experiment Station, University of Illinois (at Urbana), the Illinois Division of Highways, and the United States Bureau of Public Roads (later the United States Public Roads Administration).

The purpose of the project, as set forth at the outset was

> "* * * to make investigations and tests of reinforced concrete slabs to determine the behavior thereof under varying conditions and to develop information which will advance the art of bridge building."

The main problem envisaged was that of the theoretical analysis and tests of bridge slabs under the concentrated loads applied by heavy trucks, but the work has necessarily been extended to the action of the supporting beams, so that the investigation has actually been applied to the complete floor systems of highway bridges.

2. General Procedure

Throughout the work, a combination of analytical and experimental study has been employed. In so far as possible, it has been the practice, on any problem, first to derive analytical solutions and then, by testing structures selected for their practical importance, to study the applicability of the analysis and the deviation of actual conditions in the structure from those assumed in the analysis. The experimental program has been largely confined to laboratory tests, although some field studies and observations have been made. The laboratory structures tested have necessarily been models, generally with scales of 1:2 to 1:5. Most of these models were first designed in the full-sized prototype, using standard design methods and loadings specified by the American Association of State Highway Officials (AASHO). Great care was taken to keep the materials and dimensions of the models in true proportion to those of the prototype, and many studies were made to determine whether any scale effect existed between large-scale and small-scale models.

Obviously, the major difficulty in the analysis of the action of highway bridge slabs arises from the fact that they are subject to truck loadings. In tests, such loadings can be simulated by the application of concentrated loads, applied to represent wheel loads of trucks. In analyses, certain assumptions must be made, such as those used by H. M. Westergaard,[2] M. ASCE, in his

[1] Research Prof., Eng. Materials, Univ. of Illinois, Urbana, Ill.

[2] "Computation of Stresses in Bridge Slabs Due to Wheel Loads," by H. M. Westergaard, *Public Roads*, March, 1930, p. 1.

classic study of the effect of loads concentrated on a small finite area of a simple span slab of infinite width. Many strain measurements were made in the vicinity of such load points in the test slabs, to study the validity of the assumptions.

In the experimental work, a feature common to all the large model tests has been the study of the behavior of slabs in both the uncracked condition and the cracked condition. The former is of interest in comparing the action of the slab at low loads, before cracks have formed and when the structure most nearly approaches the elastic, homogeneous slab assumed in the theoretical analysis. At these low loads, the procedure has generally been to apply a series of loads, moved from point to point to simulate a moving vehicular load, and to make strain measurements, using sensitive gages, at points of critical stress.

It is necessary also to determine the behavior of the structure at design loads. To this end, a series of design loads was frequently applied over most of the surface of the slab—simulating the effect of continued traffic which might produce general cracking of the slab. The slab was then subjected to a series of design loads at selected points, with extensive strain measurements, to furnish load-strain data for the cracked structure. This procedure affords a picture of the expected behavior of the slab in a bridge in actual service.

A third feature of the experimental tests, is a study of conditions at or near failure, in which loads representing wheel loads were applied at critical sections of the slab to produce failure. Such loads generally reached several times the design load, and the manner of failure was observed carefully. In many cases, with multiple-lane slabs, a group of concentrated loads representing the rear wheel loads of two passing trucks was employed to produce the maximum load condition.

Since the ultimate object of the entire program is to improve the design of bridge floors, the published reports have been planned to cover three definite phases of the study: (1) The mathematical analysis, along conventional lines; (2) the experimental research on the structure, with results analyzed and correlated with the theoretical findings; and (3) the development of simplified design procedures for easy and rapid use. Phase (3) may be accomplished in many cases by fitting empirical equations to both computed and observed results, but always with the primary object of producing a simple and usable procedure for the designing engineer. This last step in the program was completed for the right, simple-span, solid-slab bridge in 1946. By the beginning of 1947, thirty-six scale-model bridges, ranging in length from 5 ft to 30 ft, and more than two hundred smaller specimens consisting of concrete or mortar slabs, composite T-beams, etc., had been tested.

3. Testing Methods

Since most of the test slabs were too large to be placed in the ordinary testing machine, some very adaptable testing rigs were built. Structural steel loading frames were erected and anchored on the heavy reinforced concrete floor of the laboratory. These frames enclosed the test slab and, in conjunction with jacks, dynamometers, and distributing beams, furnished the loading

apparatus. The frames were movable and permitted load applications at several points. The dynamometers were built and calibrated in the laboratory.

Strain measuring apparatus varied considerably over the 10-year period. The type depended on the precision required and included attached Huggenberger and graphic gages, special interferometer and level bubble gages, and the well-known Berry and Whittemore gages. On continuous I-beam bridges tested recently a large number of SR-4 electric gages were used. Deflections were generally measured by deflectometers equipped with dial micrometer gages.

4. Fundamental Studies

Several studies were made which have very general application and are not limited in application to highway bridge slabs. One of these is the analytical solution by V. P. Jensen, M. ASCE, of a large number of slab problems, involving many combinations of slabs supported on rigid or flexible beams with different degrees of continuity.[3] Moments were derived for the supporting beams as well as for the slabs. Among other cases investigated was the ordinary single-span slab with curbs.

Another extremely useful contribution to slab theory is particularly applicable to the slab supported on parallel flexible beams, as in the slab-and-stringer bridge. In this ingenious method, derived by Professor Newmark (and described in the second Symposium paper), an extension of the moment distribution method for continuous beams is made applicable to slab problems.[4] The method is equivalent to the solution of a number of analogous continuous beams, corresponding to the number of terms in the infinite series applied to the slab which are needed to give the required precision. The analysis of a slab continuous in one direction is therefore more laborious than the solution of a beam by moment distribution but the same basic principles are used. When the supporting beams are flexible, an added step in the analysis is required, somewhat like that for sidesway in a rigid frame.

Another fundamental study by Professor Jensen is concerned with the so-called plastic theory of failure of reinforced concrete beams. In the study of most of the slab results, rather large discrepancies have been found between the stresses computed by the conventional straight-line theory, assuming no tension in the concrete, and the stresses determined from the measured strains. Part of the discrepancy is doubtless due to the existence of tension in concrete, and part to a departure from straight-line distribution of compressive stress. Professor Jensen introduced the concept of a plasticity ratio in concrete, directly related to the compressive strength, and derived general expressions for determining the ultimate load in flexure on reinforced concrete beams.

The use of the plasticity ratio and of idealized stress-strain diagrams for both concrete and steel brings about the use of a trapezoidal compressive stress diagram for concrete in flexural compression. The analysis has been

[3] "Solutions for Certain Rectangular Slabs Continuous Over Flexible Supports," by V. P. Jensen, *Bulletin No. 303*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1938.

[4] "A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams," by N. M. Newmark, *Bulletin No. 304*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1938.

verified by reference to the results of a large number of beam tests, and seems to furnish a satisfactory explanation of the phenomena of beam failure.[5]

Other studies on fundamental problems include an unpublished analysis, by Professor Newmark and W. A. Renner, of slabs composed of a nonisotropic material. The application to slabs having distinctly different reinforcement in the two directions is obvious.

A similar study is concerned with the effectiveness of reinforcing steel placed in various directions in a slab. The most general application is in the skew-slab bridge, where the direction of principal stress can only be discovered by analysis or test, and where this direction varies at different parts of the slab. In such skew slabs, it is common practice to place the steel so that it makes a fairly large angle with the direction of the major principal stress. Furthermore, in such bridges, the reinforcement is frequently placed in two directions which are not at right angles to each other.

FIG. 1.—TEST SETUP OF RECTANGULAR SLAB

The effectiveness of reinforcing bars making various angles with the direction of applied moment was studied in a series of slab tests (unpublished) by Professor Newmark and John Houbolt and the results correlate fairly well with those determined by analysis.

5. EXPLORATORY TESTS

a. The Rectangular Slab, of Simple Span.—The first tests made in the concrete slab investigation were on two rectangular slabs, simply supported on two edges and subject to a single movable concentrated load.[6] The slabs were 20 ft wide, had a span of 6 ft 8 in., were 6.5 in. thick, and were loaded through a 6-in. circular disk. The width-span ratio of 3 was chosen because studies indicated that Professor Westergaard's analysis of the slab of infinite width

[5] "Ultimate Strength of Reinforced Concrete Beams, as Related to the Plasticity Ratio of Concrete," by V. P. Jensen, *Bulletin No. 345*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1943.

[6] "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads," by F. E. Richart and R. W Kluge, *Bulletin No. 314*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939.

would apply, although, for slabs of narrower width, the theory would require modifying.

The principal observations made in these tests were fairly extensive measurements of strains, deflections, and slab reactions in the region of the loads. Fig. 1 is a view of one of the slabs under test and illustrates the loading frame, dynamometer, and jack used in applying load, a series of reaction weighing rings, and a portable deflectometer bridge. Strains were measured in two directions along both center lines, and under nine different load points in the central part of the slab. The reaction rings were used only on one slab. Despite the fact that they were designed to give a deflection of only 0.002 in. at maximum load, this movement was sufficient to disturb the normal distribution of reactions and their use was omitted on later tests.

The strains at low loads, before cracking had occurred, may be studied by referring to Fig. 2. The test results for the two slabs were brought to a comparable basis by introducing the term

$$N = \frac{E\,I}{1 - \mu^2} \qquad (1)$$

which involves any variation in the quality of concrete or dimensions of the slab used in the two tests. In Eq. 1, E is the modulus of elasticity; I is the moment of inertia; and μ is Poisson's ratio. Values of N were determined experimentally by a special test of the slab, using a live load along the midspan center line of the slab. The values of N times the strain ϵ may thus be compared directly with the values computed by theory using Professor Westergaard's analysis. The four curves show the strains in top and bottom of the slab, in the x-direction and in the y-direction, on both the x-axis and y-axis of the slab. Compressive strains at the top surface were adjusted so as to apply to an element below the top which was at the same distance from the computed neutral surface as was the bottom of the slab. The adjustment was made by multiplying the measured strains by the ratio 3.13 : 3.37. In general there is a fair correspondence between experimental values, with the latter somewhat greater in the vicinity of the load point. The distribution of strains along each axis gives a diagram quite similar in shape to the theoretical distribution.

Another comparison between measured and computed strains at higher loads, after cracking has occurred, is shown in Figs. 3 and 4. Fig. 3 shows strains in the concrete at the top of the slab, whereas Fig. 4 shows corresponding values for the steel in the bottom of the slab. The reinforcement consists of $\frac{1}{2}$-in. square bars at 4-in. spacing in both directions, with an effective depth of $5\frac{1}{2}$ in. for the short bars and 5 in. for the long ones. This comparison is of interest, since it shows the relation between the measured concrete strains and values computed both considering and neglecting the tension in the concrete. At the 16-kip load the measured strains fall between the two computed curves, but at the 48-kip load the measured values agree best with the curve based on no tension in concrete. A somewhat different effect is noted from examination of the steel strains in Fig. 4 in which the steel strains, even at the highest load shown, are generally no more than 80% of the computed values, and are less

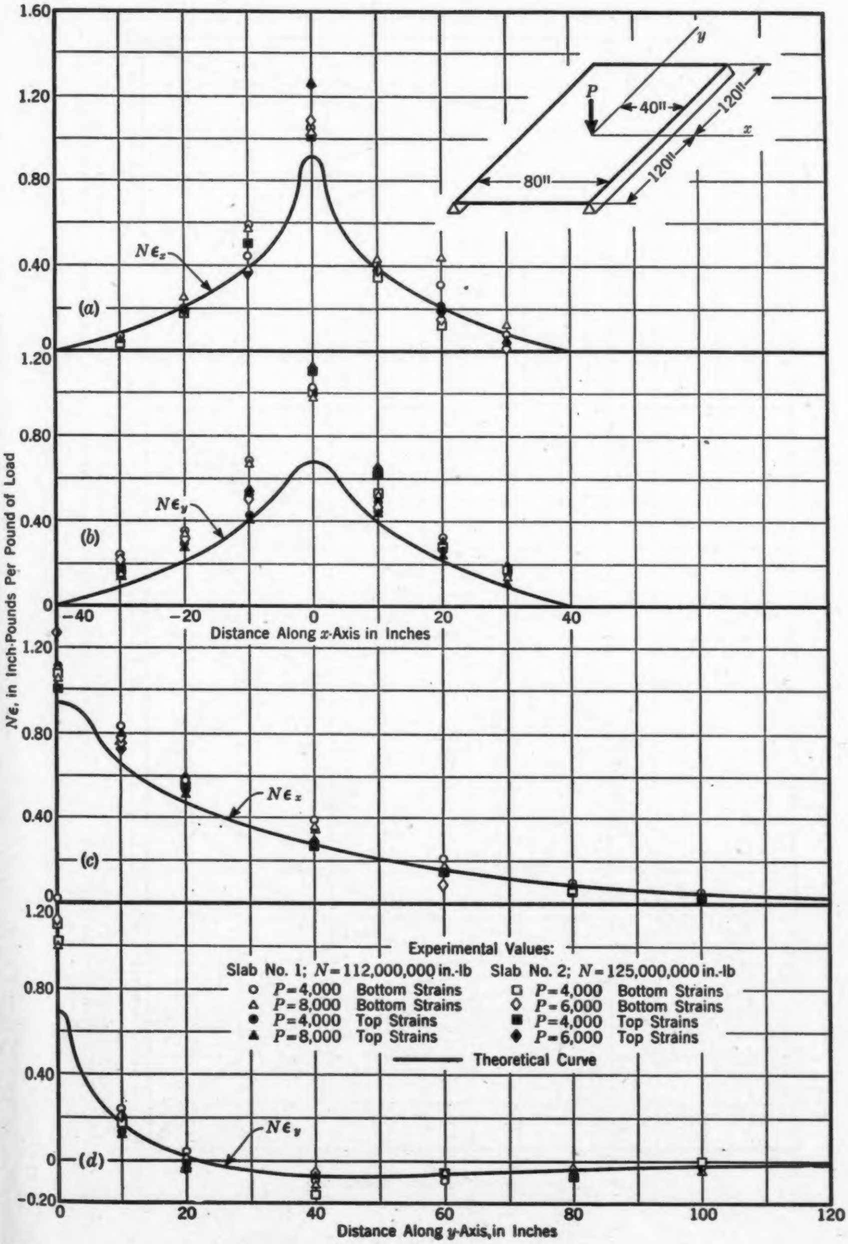

FIG. 2.—WEIGHTED STRAINS ($N\ \epsilon$) RESULTING FROM LOW LOAD AT MIDSPAN

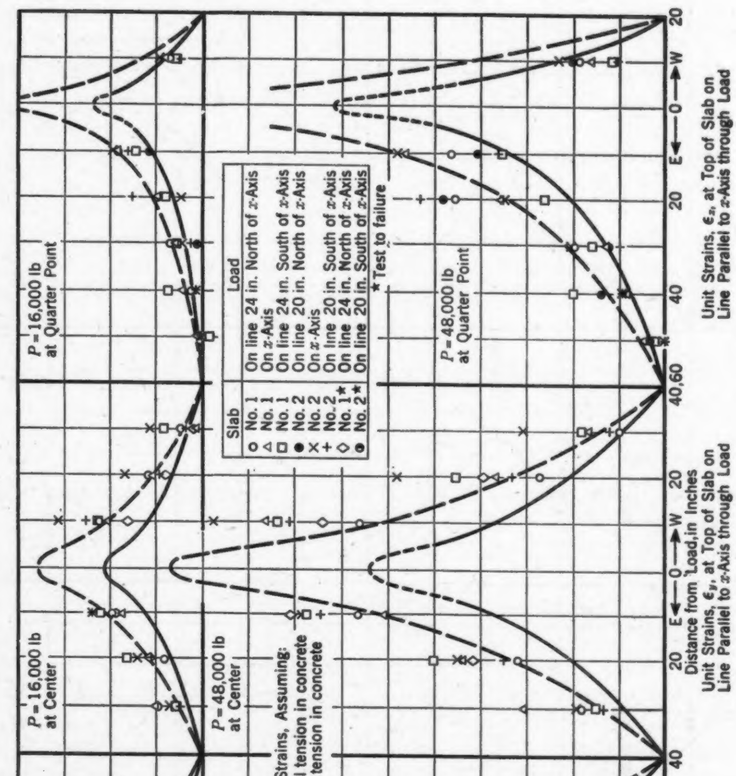

FIG. 3.—CONCRETE STRAINS IN SLAB DUE TO LARGE LOADS

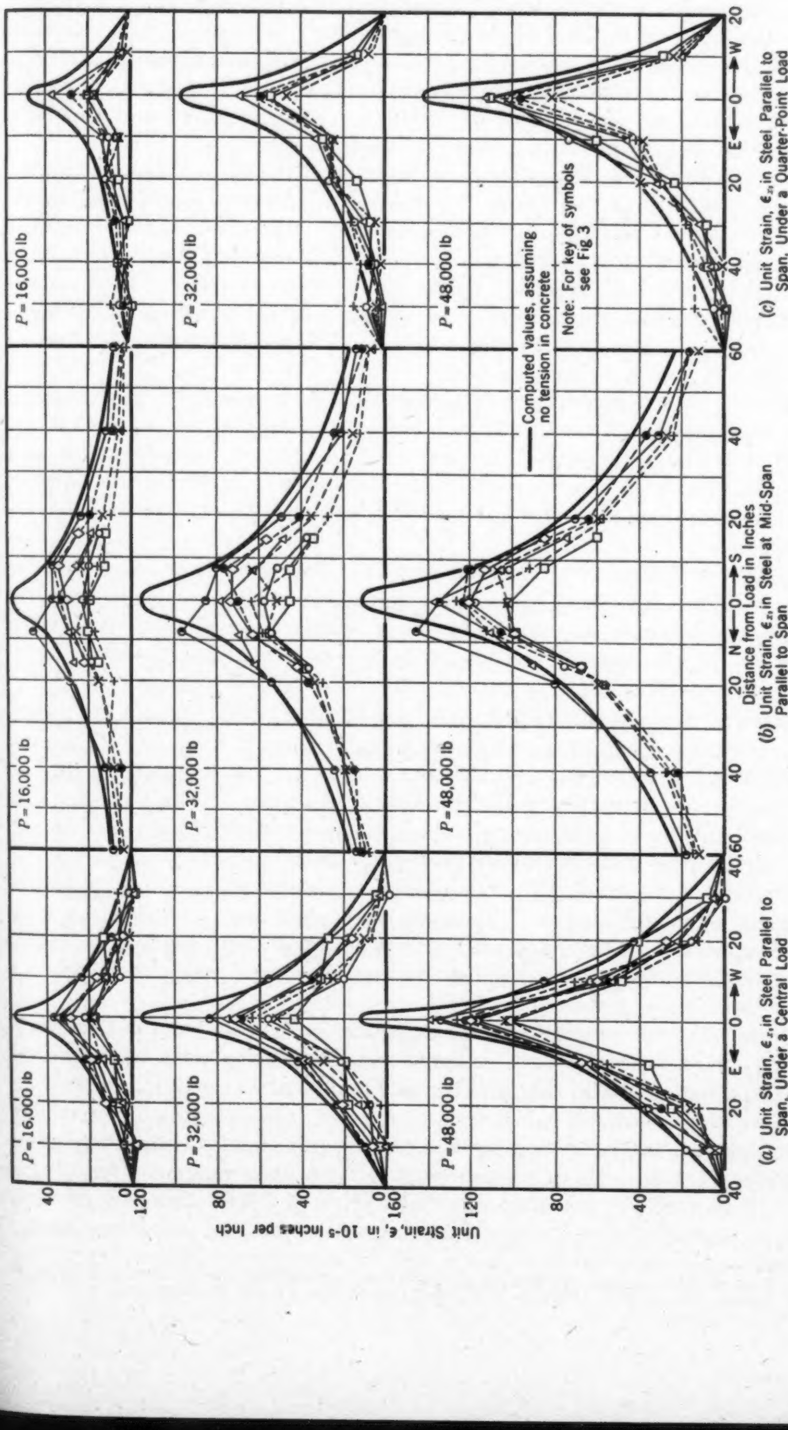

FIG. 4.—STEEL STRAINS IN SLAB DUE TO LARGE LOADS

than this directly under the load. These comparisons involve an inconsistency, since the moments in the slab were computed for a homogeneous slab, whereas the strains in the steel are based on the conventional assumption that no tension exists in the concrete. However, as an exploratory test, these studies served to indicate the kind of deviations between measured and computed values that have been found in many of the later tests.

In tests to failure, a final punching through of the slab occurred under the loading disk, but only well after the yield point of the steel had been passed and general cracking of the slab had developed. This action was also observed in practically all the subsequent tests of slabs under concentrated loads. By applying the load to widely separated points on the slab, several failures were produced, with average values of 83.5 kips for slab No. 1 and of 79.6 kips for slab No. 2. The location of the load point apparently had little effect on the value of this final punching load.

It may be of interest to note that quarter-scale models of the two large slabs were made and tested. These were mortar slabs, with ⅛-in. bars, fairly true to scale in all respects. Because of their small size, a precise strain gage was needed and an interferometer extensometer was built for the purpose. The results of these model tests compared remarkably closely to those from the large slabs.

b. Square Slabs, with Concentrated Load Applied Over Various Bearing Areas.—When these tests were made, there was much speculation over the proper application of concentrated loads to simulate the effect of truck wheel loads. Since that time several studies have been reported on the shape of the tire contact and the pressure distribution within that bearing area.

A series of twenty square slabs, 5 ft wide and 5 ft in simple span, was tested, using central bearing areas as follows: Circular disks, 2 in., 6 in., 10 in., and 14 in. in diameter; rings, 1 in. thick and 6 in. and 14 in. in outside diameter; and pairs of circular disks, 2 in. and 6 in. in diameter.[7] The 6-in. disk was used with a layer of either sponge rubber or plaster of Paris between disk and slab; for the other tests, the sponge rubber was used.

Because of the small size of the slabs, they were tested in a large testing machine, although jacks and dynamometers were used to apply the load. Compressive strains were measured with the graphic strain gage, by providing a suitable groove in the loading disk, and tensile strains were obtained with a Berry gage.

Both the compressive and tensile strains under the load point of these slabs showed almost constant values for the five smaller bearing areas, whereas those for the 14-in. disk, 14-in. ring, and the pair of 6-in. disks, at 12-in. center spacing, gave definitely lower values for a given load. This condition is shown for the steel strains in Fig. 5 where the two definite groups of load-strain curves are evident. With so little difference resulting from disks ranging from 2 in. to 10 in. in diameter, it was concluded that the use of the circular disk, of a size properly related to the size of the truck tire, was justified for subsequent tests.

[7] "Tests of Reinforced Concrete Slabs Subjected to Concentrated Loads," by F. E. Richart and R. W Kluge, *Bulletin No. 314*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939, p. 51.

As in the previous series of tests, these tests on loading areas were repeated in quarter-scale models of reinforced mortar. The results corroborated those from the larger slabs.

c. Tests of Plaster Models.—As one of the early studies of slab action under concentrated loads, a fairly large number of plaster-of-Paris model slabs were

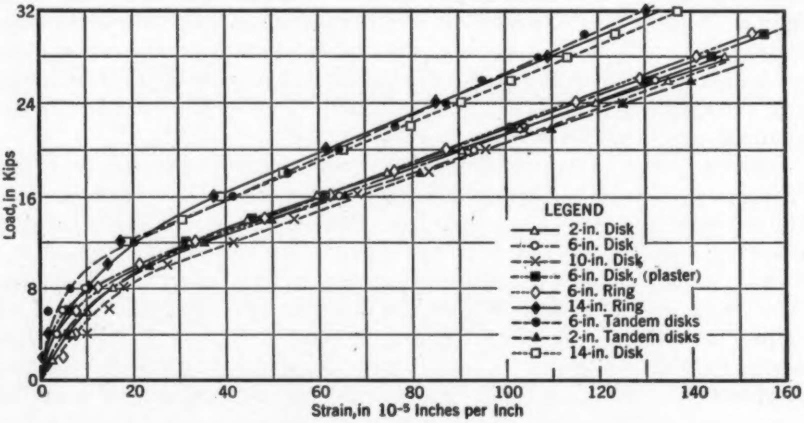

Fig. 5.—Load-Strain Curves for Reinforcement Directly Under the Load

made and tested.[8] Since the ordinary theory of flexure is not valid at or near the load point, and since the stress-strain relation for pottery plaster is a fairly straight line up to the ultimate load, it was reasoned that test slabs of this material would furnish a good indication of the stresses existing in an elastic slab and thus would supplement the theoretical analysis.

Four groups of slabs were made, generally 1 in. thick and 12 in. in span. These included (1) circular slabs, supported around the periphery, with variable loading area and variable thickness; (2) rectangular slabs, supported on two edges, with variable width and load position; (3) rectangular slabs, supported on four edges, with variable load position and restraint of corners; and (4) miscellaneous square slabs, with support at corners. The results of these tests were very useful. Where the stresses were computed by the theory of elasticity, the measured results showed fairly good agreement. There was some indication that, although control beams showed a straight-line stress-strain curve, some plastic adjustment occurred in the slabs under the load concentration. In general, for slabs of comparable size, the material failed according to a limiting tensile flexural stress.

The tests gave valuable information on the effect of size of bearing area, and corroborated other findings that the exact shape of the loaded area is not of major importance. The ratio of bearing area to slab thickness is much more significant. The type of support at the edge of a slab does not have much effect on the stresses near a central load, and a load near a support is of greater importance in a slab than it would be in a beam.

[8] "Tests of Plaster-Model Slabs Subjected to Concentrated Load," by N. M. Newmark and H. A. Lepper, Jr., *Bulletin No. 313*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939.

6. Simple-Span Right-Slab Bridges with Curbs

The simple slab used in highway bridges is provided with a curb and handrail which considerably modify the stress distribution that exists in a slab of uniform thickness. This type of bridge finds such widespread use that it was one of the first to be studied in the concrete slab investigation. The mathematical solution of the slab with stiffened edges has been presented by Professor Jensen.[3,9] On the basis of this analytical work, laboratory tests were made on two half-scale and seven quarter-scale models. The principal variables studied were the span length and the relative stiffness of curb and slab. In one test, handrails were included to determine their effect.

Except for curb details, these models were geometrically similar to full-sized bridges, designed by AASHO specifications, for H-20 truck loadings. The concrete used had a compressive strength varying from 3,500 lb per sq in. to 4,000 lb per sq in. and the reinforcing steel was of intermediate grade.

As in other slab tests, the strains measured in these models agreed quantitatively in some of the tests and only qualitatively in others. In general, the strains measured in the principal (longitudinal) reinforcement were a little smaller than the computed values, whereas the strains in the transverse reinforcement were very much less than those computed. In several cases, the compressive strains at midspan of curb were unduly high in spite of the fact that much more compressive reinforcement had been used in the curbs than is

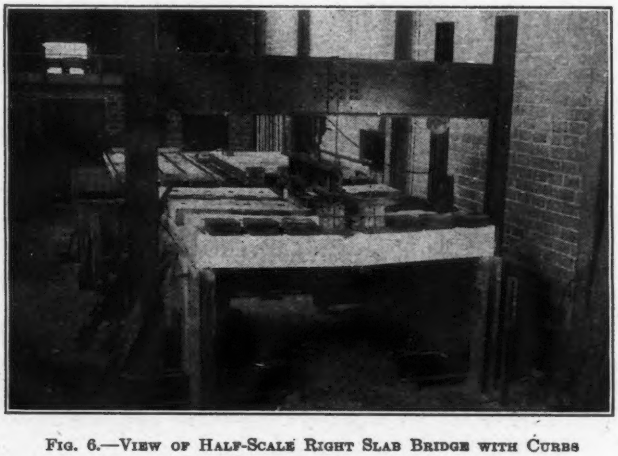

Fig. 6.—View of Half-Scale Right Slab Bridge with Curbs

common design practice.

A striking result of the tests was the high strength of the slab, as related to the design load. In most of the tests, the critical load (where it was evident that yielding of the longitudinal steel had occurred near midspan) was equal to the dead load plus from 4 to 5 live loads. Beyond this point there was a

[9] "Moments in Simple Span Bridge Slab with Stiffened Edges," by V. P. Jensen, *Bulletin No. 315*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1939.

further reserve of strength of about 25% beyond the critical load. Such bridges evidently have a high factor of safety.

Fig. 6 shows one of these large models under test, with apparatus for applying four concentrated loads, representing two sets of rear truck wheels.

The results of these studies have been presented elsewhere by Professor Jensen, with R. W. Kluge, Assoc. M. ASCE, and C. B. Williams, Jr., Jun.

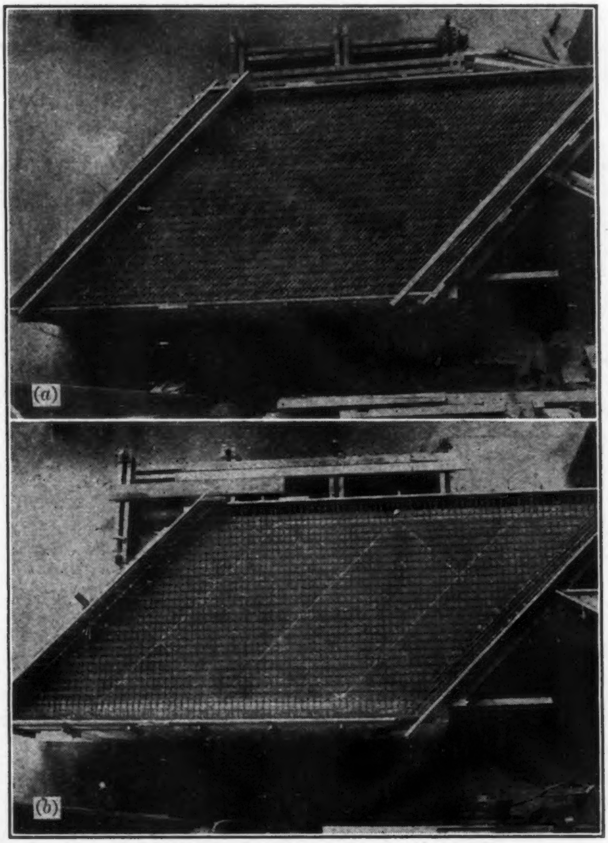

Fig. 7.—Reinforcement of Simple-Span Skew-Slab Bridges

ASCE. Their study[10] presents the final objective of the work, a simplified method of design. The procedure is intended to cover the usual bridges of this type, with spans of 30 ft or less, but also treats other cases, such as bridges of longer spans or those with unusual types of curb.

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[10] "Highway Slab-Bridges with Curbs: Laboratory Tests and Proposed Design Methods," by V. P. Jensen, R. W. Kluge, and C. B. Williams, Jr., *Bulletin No. 346*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1943.

7. Simple-Span Skew-Slab Bridges with Curbs

Since many slab bridges are built as skew spans, it was necessary to extend the studies of Section 6 to the skew slab. The mathematical analysis of the skew slab under concentrated loads is fairly difficult, but Professor Jensen developed a practical procedure involving the solution of numerical cases by the use of difference equations.[11] Although the solutions involved some degree of approximation, they are believed to be the only answer to the problem available. In Professor Jensen's report a careful study is made of the boundary conditions and of the effectiveness of reinforcing steel when placed in two directions generally not at right angles. Applying these methods to the present study, solutions have been made for a number of typical bridges of varying

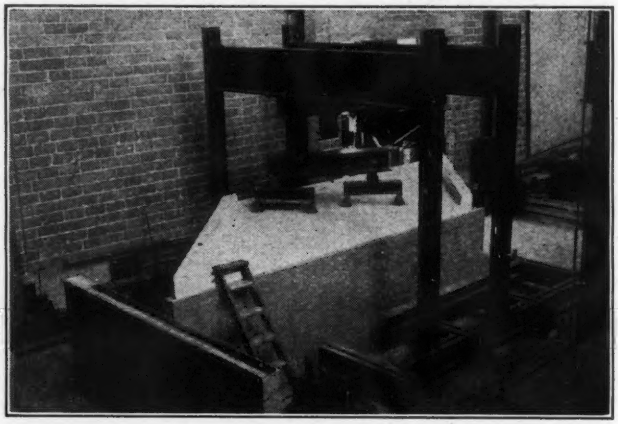

Fig. 8.—View of Half-Scale 45° Skew Slab with Curbs

proportions and skew angles. Influence surfaces for moments in these bridges have been determined and, on the basis of the results found, curves have been plotted which may be used in the design of bridges within the range of the proportions studied. The method, as used thus far, has not reached the stage of simplification for design practice that has been developed for right bridges.

The foregoing analysis was used, however, to design three half-scale models of 45° skew bridges and one one-fifth scale model of a 60° skew span. The three half-scale models present an interesting study of the arrangement of the reinforcement. In model No. 1, the main or longitudinal reinforcement is placed parallel to the roadway (see Fig. 7(*a*)) despite the fact that analysis indicates that the principal stresses in the central part of the bridge are in a direction normal to the abutments.

Because of the ineffective angle, the amount of this main reinforcement is twice as great as is required in model No. 2 (see Fig. 7(*b*)), in which the main steel is placed normal to the abutments. In model No. 3, the steel is placed in the same direction as in model No. 1, but in reduced amount, with a view to

[11] "Analysis of Skew Slabs," by V. P. Jensen, *Bulletin No. 332*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1941.

determining whether such a design would produce a satisfactory factor of safety. A view of one of these bridges under test is shown in Fig. 8.

As in the case of the right bridge slabs, the factors of safety for models No. 1 and No. 2 were quite large. The load carried at first yielding of the reinforcement, in addition to the dead load, was 3.5 live loads for model No. 1, 4.0 live loads for model No. 2, and 2.4 live loads for model No. 3. Corresponding numbers of live loads at failure were 7.5, 7.25, and 4.7, for the three models, respectively.

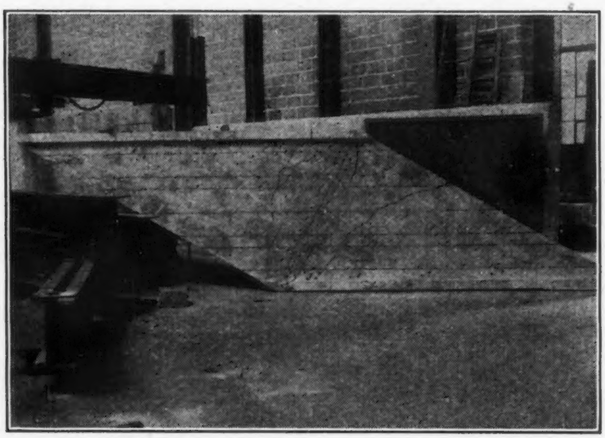

FIG. 9.—CRACKS IN THE TOP SURFACE OF A 60° SKEW SLAB

An interesting feature of a test of the one-fifth scale bridge with 60° skew (shown in Fig. 9) is the large crack that developed roughly on a line joining the obtuse corners of the slab, as produced by simulated truck loadings. Although such cracking on the top surface of the slab would generally not be anticipated, a little study indicates that its occurrence is perfectly natural.

8. THE SLAB-AND-STRINGER BRIDGE

One of the most common types of highway bridge, for spans of from 20 ft to 80 ft, is the stringer bridge, consisting of a slab supported on several steel or reinforced concrete stringers. Probably the most common case is that with five stringers, although three or four are sometimes used.

The distribution procedure developed by Professor Newmark[4] formed the basis of an extensive series of computations of beam and slab moments in some fifty bridge designs, in which variations in span and in relative stiffness of stringer and slab were systematically determined. This study covers only the bridge having five stringers. These results were analyzed and plotted, and as a final step the significant design factors were expressed in the form of empirical equations applying to the entire range of designs covered.[12]

A series of tests of I-beam bridges was made on the basis of this analytical work. These tests were all on quarter-scale models, with five I-beam stringers,

[12] "Moments in I-Beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1942.

supporting a reinforced slab continuous in the direction normal to the beams.[13] In six of fifteen right simple-span models, the slabs were mechanically anchored to the beams to provide composite action. Two spans of the prototype were used—20 ft and 60 ft—and variations were studied in the amount of reinforcement and in the effect of shear connectors. The principal objectives of the tests were to compare computed and measured strains in beams and slab, to determine the ultimate capacity of such bridges, and to aid in the development of improved design procedures.

A view of the stringers used in two of these bridges is given in Fig. 10. The shear connections used, consisting of short sections of bar channel welded to the top flanges of the I-beams, are shown in the right-hand bridge. The method of applying and measuring the load on these bridges was similar to that used on the slab bridges.

The skew I-beam bridge was also studied. However, the skew span with flexible stringers is so difficult to analyze that the test program was planned without having any analytical treatment as a guide to the experimental work. Five quarter-scale models of 60-ft I-beam bridges, with skews of 30° and 60°, were tested. Three of these bridges were designed to produce composite action of beam and slab.

Besides the tests of right and skew simple spans, three tests on two-span continuous stringer bridges have been made. Of these quarter-scale bridges, one was built with no shear connectors, the second with shear connectors throughout, and the third with the shear connectors used only in the region of positive moment.

A view of one of these bridges under test is seen in Fig. 11. It shows the application of load near the middle of each span, in position to produce a maximum negative moment at the middle support. The wiring for electric

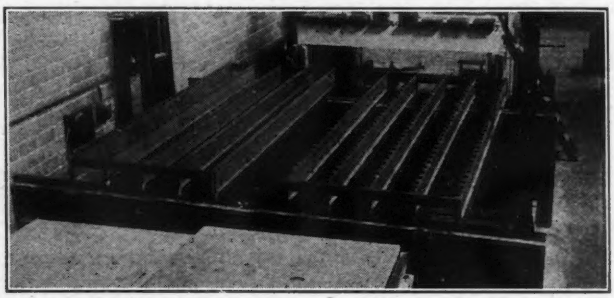

FIG. 10.—STRINGERS USED IN RIGHT I-BEAM BRIDGES

gages is seen at the top of the slab, at the middle pier, and at midspan, and leads to the switching apparatus and portable strain indicator at the side of the structure. More than three hundred electric gages were used on each of these test structures. The spans illustrated were each of 15 ft.

[13] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Seiss, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946.

9. General Comments

In addition to the main results cited in this paper, the concrete slab investigation has produced many valuable by-products, including the development of many measuring instruments and procedures.

Not the least of the benefits from the program has been the training of young men in structural research methods and techniques. During the period of its existence the investigation has had the services of ten full-time investigators

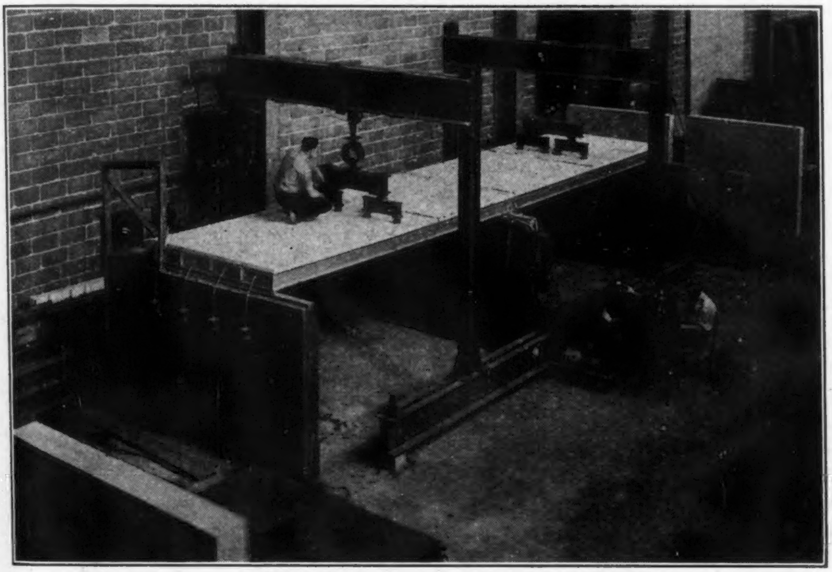

Fig. 11.—View of Two-Span Continuous I-Beam Bridge

and fifteen research graduate assistants, the latter being graduate students employed half time for a 2-year period. There have also been a number of laboratory assistants, mechanics, and student and office helpers. Although some progress has been made in solving important problems of highway bridge design, it is evident that far more remains to be done than can ever be accomplished by any single organization. It is hoped that this Symposium will serve to stimulate interest and activity in the highway bridge field which will lead to definite improvements in design practice.

10. Acknowledgments

Major credit for initiating the program belongs to Alfred Benesch, then engineer of grade separations, Illinois Division of Highways, who realized the need for more information on many features of bridge design. Others, including E. F. Kelley and A. L. Gemeny of the United States Public Roads Administration and W. M. Wilson, H. M. Westergaard, and Hardy Cross, Members, ASCE, made important contributions to the initial outline of the program. Throughout the investigation, the planning of the various projects undertaken

has been under the direction of an advisory committee, composed, in 1947, as follows:

U. S. Public Roads Administration—E. F. Kelley and Raymond Archibald, M. ASCE.

Illinois Division of Highways—George F. Burch, M. ASCE, and L. E. Philbrook, Assoc. M. ASCE.

University of Illinois—F. E. Richart and N. M. Newmark, Members, ASCE, with W. M. Wilson and T. C. Shedd, Members, ASCE, as consultants to the committee.

In addition to Professor Newmark and the writer, four technical investigators are engaged on this project at the University of Illinois: C. P. Siess, Assoc. M. ASCE, special research assistant professor; M. L. Gossard, Jun. ASCE, and W. E. Johnson, special research associates; and L. W. Jones, research graduate assistant. Two laboratory mechanics and student and clerical assistants also compose an essential part of the project.

The investigation is under the administrative direction of M. L. Enger, M. ASCE, dean of the College of Engineering and director of the Engineering Experiment Station, and F. B. Seely and W. C. Huntington, M. ASCE, respective heads of the departments of Theoretical and Applied Mechanics and Civil Engineering at the University of Illinois.

DESIGN OF I-BEAM BRIDGES

BY N. M. NEWMARK,[14] M. ASCE

1. INTRODUCTION

Recommendations are presented herein for the design of the slab and the beams of I-beam highway bridges. Consideration is given to simple and continuous spans, to right and skew bridges, and to bridges in which T-beam action is obtained by the use of either shear connectors or concrete beams. The treatment is based on both analyses and tests. The practical significance of the design recommendations is discussed in the light of current practice and current specifications.

The I-beam bridge, consisting of a concrete roadway slab continuous over steel stringers, has found widespread application for highway bridges because of its simplicity of design and ease of construction. This paper discusses the behavior of the slab-and-stringer bridge having beams in the direction of traffic. The slab may be mechanically bonded to the beams by shear connectors. The bridges considered, in general, are supported by four or more parallel beams of equal stiffness at a constant spacing of the beams between the limits of 5 ft and 8 ft.

The terminology used in the paper is illustrated in Fig. 12. The span of the bridge is denoted by a, the spacing of the beams by b, and the thickness of the slab by h. The main reinforcement in the slab is in the direction transverse to the beams; the longitudinal reinforcement is secondary reinforcement.

The analytical background for this work was presented by the writer in 1938[4] and in 1942.[12] The experimental background is based mainly on the writer's tests of quarter-scale models at the University of Illinois.[13,15] A paper on the same general subject, but with more conservative recommendations based almost entirely on analyses, appeared in 1943.[16] The analytical studies[4,12] involve the usual assumptions of the ordinary theory of flexure of slabs. The detailed calculations are long and tedious. However, influence coefficients for moments in the slab and in the beams at various points for a number of structures of different proportions are summarized in the 1942 report.[12] Simple empirical equations expressing the moments resulting from truck loads in both the slab and the beams are also given in this report.

As stated, the experimental work[13,15] has been entirely on quarter-scale models. Initial studies indicated that duplication of the behavior of full-scale prototype structures could be obtained in quarter-scale models if attention were paid to the proper scaling of the aggregate and reinforcement. Extensive studies indicated good correlation, over the entire range of action, from elastic

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[14] Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

[15] "Studies of Slab and Beam Highway Bridges: Part II. Tests of Simple Span Skew I-Beam Bridges," by N. M. Newmark, C. P. Siess, and W. M. Peckham, *Bulletin No. 375*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1948.

[16] "Design of Slab and Stringer Highway Bridges," by N. M. Newmark and C. P. Siess, *Public Roads*, January–February–March, 1943, pp. 157–164.

behavior at low loads, the subsequent action at working loads, and the behavior after considerable yielding of the reinforcement, to and including the failure of the slab by punching. The reinforcement used in the models consisted of bars, ⅛ in. square, specially heat treated to have stress-strain characteristics corresponding to the full-sized reinforcement used in an actual structure.

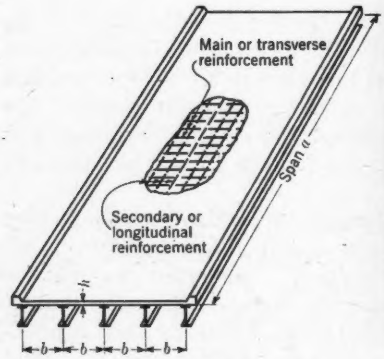

FIG. 12.—TYPICAL I-BEAM BRIDGE

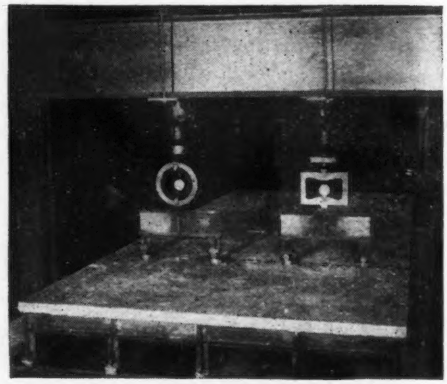

FIG. 13.—QUARTER-SCALE MODEL OF I-BEAM BRIDGE

Fig. 13 shows a completed model bridge subjected to tests in the laboratory.[17] The loading represents the rear wheel loads of two trucks in adjacent lanes. Tests have been made on both right and skew bridges, with and without shear connectors, and on continuous bridges.

2. DISTRIBUTION OF WHEEL LOADS TO SLAB AND STRINGERS IN SIMPLE-SPAN RIGHT I-BEAM BRIDGES

a. Fundamental Concepts.—The moments in the slab and in the beams of an I-beam bridge are dependent on the geometric proportions of the bridge and on the stiffness of the beams relative to the stiffness of the elements of the slab in a direction transverse to the beams. The "effective width" of the slab, for the purpose of computing relative stiffness, is dependent on the type and arrangement of loading and on the beam spacing. In previous analytical work it was convenient to define the stiffness of the beams relative to that of a width of slab equal to the span of the bridge, although the latter is greater than the "effective width" for most practical conditions. In order to keep the same definition in this paper, the relative stiffness of the beams, denoted by the symbol H, is defined as the dimensionless ratio of the product of the modulus of elasticity, E_b, and the moment of inertia, I_b, of the beam to the corresponding product for a width of slab equal to the span of the beams:

$$H = \frac{E_b\, I_b}{a\, E\, I} \quad \text{.......................(2)}$$

[17] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 25, Fig. 5.

In Eq. 2, E is the modulus of elasticity of the slab; and I is the moment of inertia per unit of width of the slab, which can be taken as $h^3/12$.

The quantity of greatest influence on the action of the slab in distributing loads to the beams is the stiffness of the slab in the direction transverse to the beams. However, the slab also has stiffness in the longitudinal direction and consequently carries part of the longitudinal moment. The part of the longitudinal moment carried by the slab is small, and at any section through the bridge where the slab may be cracked the beams carry a relatively larger share. It is on the safe side, and not unreasonable, to assume that the beams carry all the longitudinal moment.

The effect of the relative stiffness of the beams on the moments resulting from a single concentrated load is large; but the effect on the moments caused by truck loads (which control the design of the I-beam bridge) is small. This is fortunate because the magnitude of H is subject to considerable uncertainty, since its value depends on the modulus of elasticity and on the moment of inertia of the concrete slab.

In general, the value of H for representative designs of a 60-ft span may range from 3 to 8 for noncomposite beams, from 5 to 15 for composite I-beams, and from 15 to 30 for concrete T-beams. For other span lengths, the values of H are roughly proportional to the span, increasing slightly less rapidly than the span increases.

When the slab cracks from moments in the transverse direction, the relative flexural stiffness of the slab at the crack is reduced. However, the average stiffness of the slab determines the distribution of the loads to the beams, and the average transverse stiffness is only slightly reduced by cracking since there are usually fairly large sections of the slab that remain whole between cracks. For this reason, the value of I used in defining H can be taken as the value corresponding to a plain concrete slab of full depth h, without cracks. The slight increase in stiffness resulting from the presence of the transverse steel is balanced to some extent by the slight decrease caused by cracking. The results of tests justify this choice of I for the slab in the determination of H.

Since H varies inversely as the cube of the depth of the slab, the depth of the slab has great influence on the action of the bridge. For example, changing the depth from 6.5 in. to 7 in., other factors being equal, will reduce H by very nearly 20%. For a given type of bridge, however, the range in values of H is generally small enough that formulas for the moments controlling the design can usually be made independent of H.

The wheel loads considered herein are those for the standard H-truck loading in the AASHO Standard Specifications for Highway Bridges.[18]

The standard H-truck has the wheels of each axle spaced 6 ft apart, with front and rear axles spaced 14 ft apart. Each of the rear wheels carries a weight of four tenths of the total weight of the truck, and each of the front wheels carries a weight of one tenth of the total weight of the truck, or one fourth of the rear wheel weight. The weight of the truck, in tons, is designated by a numeral; thus: H-20. The rear wheel load P, in terms of which

[18] "Standard Specifications for Highway Bridges," AASHO, Washington, D. C., 4th Ed., 1944.

moment coefficients are stated, is the weight on a rear wheel increased by the impact factor.

Each truck is considered to occupy the central part of a 10-ft traffic lane; therefore, the minimum distance between the center of a wheel and the face of a curb is taken as 2 ft, and the minimum distance between centers of wheels of trucks in adjacent lanes is taken as 4 ft.

A subsequent revision in the specifications has the effect of considering a single maximum wheel-load concentration of 12,000 lb, corresponding to an H-15 truck, in the design of the floor slab for a bridge designed for H-20 loading.

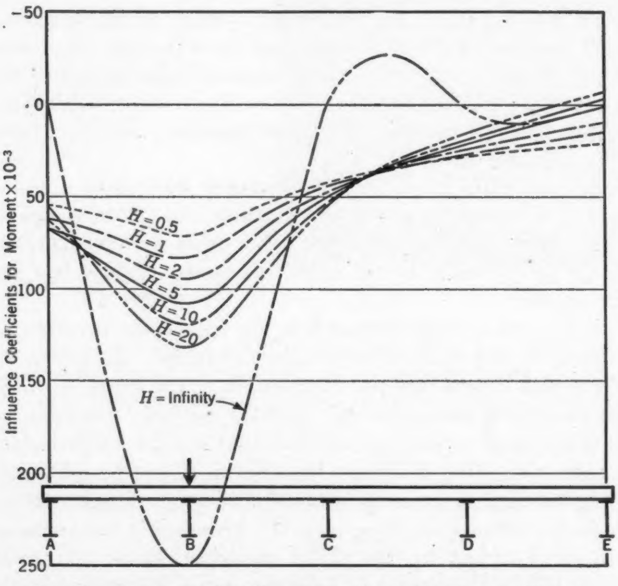

FIG. 14.—EFFECT OF SLAB STIFFNESS ON THE MOMENTS IN BEAMS, FOR A SINGLE LOAD IN THE BRIDGE

b. Action of Slab in Distributing Loads to Beams.—The principal load-carrying elements of an I-beam bridge are the beams themselves; the slab is merely the roadway, and can be repaired without serious difficulty if it is locally damaged. However, the loads reach the beams through the slab, and the magnitude of the loads carried by the beams is affected to an important extent by the stiffness of the slab. As an indication of the role the slab plays in distributing loads to the beams, consider Fig. 14 which shows influence lines for moments at midspan of beam B, an intermediate beam in a 5-beam bridge, in which $b/a = 0.1$. By reason of a reciprocal relation, the ordinates to the diagram at points under the beams show also the moments in all beams for a single concentrated load at beam B.

The stiffness of the slab is measured by the reciprocal of the value of H. For a slab of zero stiffness, $H = \infty$, and the moments in all beams, except beam B, are zero, whereas the moment in beam B is $P\,a/4$. On the other hand

for $H = 0.5$ the moment in beam B is only $0.071\ P\ a$, and the moments in the adjacent beams are not much less. This fact emphasizes the importance of the slab thickness in distributing the loads to the beams.

To a marked extent the slab acts in the same manner as a diaphragm or a transverse truss. The major difference between the action of the slab and that of a diaphragm is in the nature of the loading transferred to the beams. For a diaphragm, the loads carried to the beams are essentially concentrated loads at the points where the diaphragm crosses the beams. The nature of the loading transferred to the beams by the slab is shown in Fig. 15. The complete moment diagram for each beam is indicated for a unit concentrated load on beam B. The nature of the moment diagrams is noteworthy: For beams A, C, D, and E, the curve is roughly a sine curve; but, for beam B, it is concave

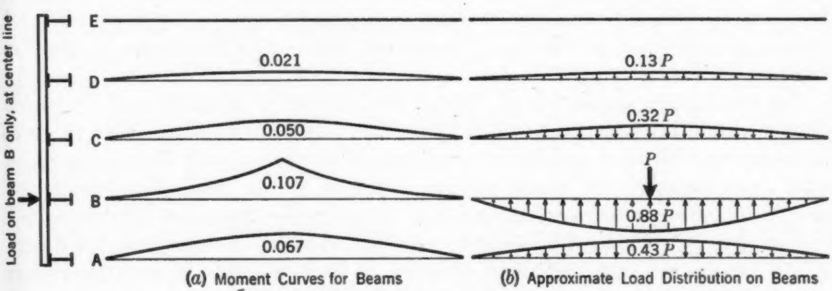

FIG. 15.—ACTION OF SLAB IN DISTRIBUTING CONCENTRATED LOAD, AND THE RESULTING MOMENTS IN BEAMS ($H = 5$ AND $b/a = 0.1$)

upward and has a cusp at the load point. Also shown in Fig. 15 are the loading curves that correspond to the moment diagrams. The loadings are approximately sine curves and are downward for all beams except B. For beam B the loading consists of two parts—a downward concentration equal to P, and an upward sine curve of distributed load. The total distributed load is indicated on each curve.

The fact that tensions between the beam and the slab arise in the I-beam bridge results from the difference in the nature of the deflections of a slab and a beam subjected to concentrated loads. The deflection curve is more localized for a slab, but is smoother for a beam. Hence, the two elements tend to separate. Separation is furthermore caused by the tendency of the slab to curl because of shrinkage and temperature variations.

The action of the slab as a diaphragm or as a transverse beam can be considered as follows: For a line load distributed over the length of the bridge approximately as a sine curve, the slab acts as a transverse beam with an effective width practically equal to the span length. For a concentrated load at midspan, the first term in the Fourier sine-series expansion accounts for about 81% of the moment at midspan. The remaining terms in the series, accounting for the remaining 19% of the moments, are distributed in different ways with different effective widths of the slab. The net effect is practically the same as if the slab were considered as a diaphragm having an effective width somewhat

less than the length of the span, but more than half of the span. The exact proportion does not make much difference as the distributions of loading to the beams are not greatly sensitive to minor variations in stiffness of the slab or of transverse diaphragms. One possible diaphragm analogy, in which the effective width of the slab is taken as half the span for 81% of the load, and the remaining 19% is considered as being carried by the loaded beam, gives results which are on the safe side, and in excellent agreement with more elaborate analyses.

Since the slab acts as a very effective diaphragm, it is unnecessary to provide additional diaphragms, except for construction purposes, if the slab can perform its function of distributing loads to the beams at the same time that it provides a roadway for the wheels to roll over. Where it is expedient or desirable to make the slab thin, and therefore flexible, some additional transverse bridging is desirable. In general, however, such bridging is not particularly effective except for loads at or close to the section where the transverse frames are located.

As the loading on an I-beam bridge approaches in magnitude the capacity of the bridge, both the beams and the reinforcement in the slab start to yield. Although one element may yield before another, the over-all effect is to maintain approximately the same relative stiffness of the beams; consequently, even near capacity loads, there are differences in the moments carried by the various beams. It is not necessarily safe, therefore, to use as a basis for design of the beams the so-called plastic theory of limit design. Such a procedure would be applicable and justified only when the slab or other transverse framing remained stiff enough at the higher loads to distribute the loads equally to all the beams.

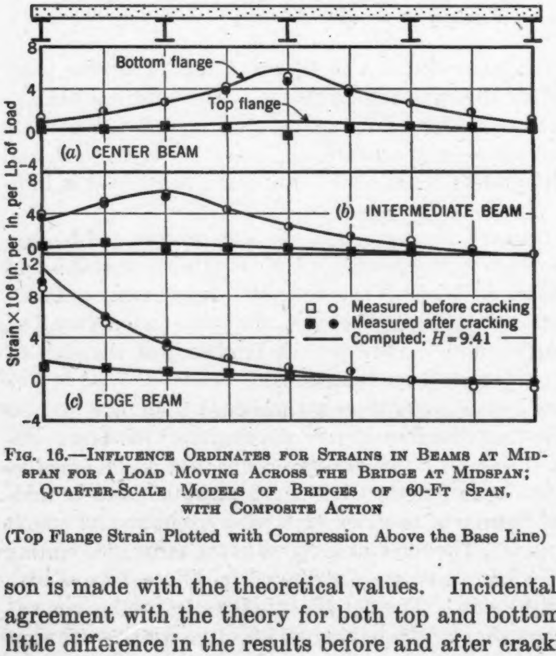

FIG. 16.—INFLUENCE ORDINATES FOR STRAINS IN BEAMS AT MIDSPAN FOR A LOAD MOVING ACROSS THE BRIDGE AT MIDSPAN; QUARTER-SCALE MODELS OF BRIDGES OF 60-FT SPAN, WITH COMPOSITE ACTION

(Top Flange Strain Plotted with Compression Above the Base Line)

c. Moments in Beams.—Test results indicate good agreement between the measured and computed values of moments in the beams. As an example of the agreement, experimentally determined influence lines for flexural strains in the beams of a composite I-beam bridge are shown in Fig. 16, where comparison is made with the theoretical values. Incidentally, Fig. 16 illustrates good agreement with the theory for both top and bottom flange strains, and shows little difference in the results before and after cracking of the concrete.

When several loads are on the bridge, corresponding to trucks in adjacent lanes, the moments in the beams are much more uniform than those for a single load on the bridge, even when the relative stiffness of the slab is fairly small. Nevertheless, there is still some difference in the moments for a given arrangement of the loads. This condition is shown by Fig. 17, which is a comparison of measured and calculated beam strains in a model bridge (60-ft span without composite action) for a condition of loading corresponding to maximum moment in beams B and C. A slightly altered arrangement of loads would be required

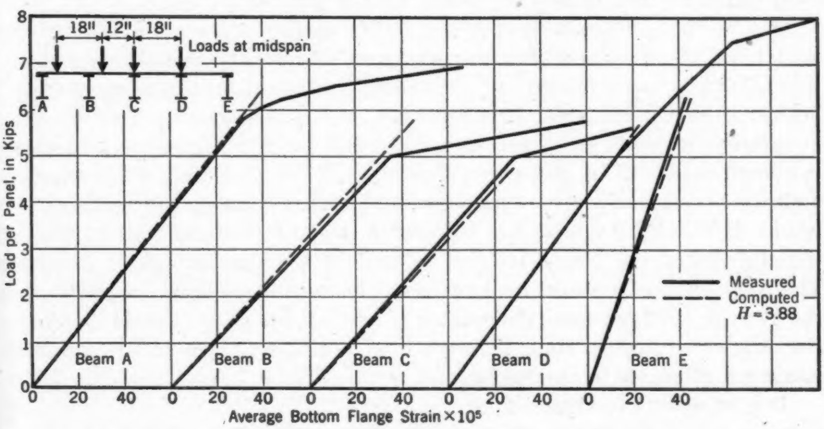

FIG. 17.—LOAD-STRAIN CURVES FOR BEAMS; QUARTER-SCALE MODELS OF 60-FT SPAN, WITHOUT COMPOSITE ACTION

to produce maximum moment in beam A unless the curb were directly over beam A. The close agreement between the measured and calculated values is to be noted.[19] For this loading, beams B and C yielded first at a panel load of about 5 kips, whereas beam A did not yield until a load of about 6 kips was reached. Beam D carried a larger proportion of load after the other beams yielded, and began to yield at a load of about 7 kips, whereas beam E did not yield at all, although the remaining beams had developed large permanent deflections and were unserviceable when the test was stopped.

The close agreement between the measured and computed beam strains is typical of the test results on long-span bridges for all arrangements of loading.[13] However, slight discrepancies were obtained in some of the tests of models of prototype bridges 20 ft in span length. The differences are easily explained. In short-span bridges, from about 20 ft to 30 ft long, the relative longitudinal stiffness of the slab is fairly high and an appreciable part of the total longitudinal moment is carried by the slab. In general, after the slab is cracked, the agreement between measured and computed maximum strains in the beams is not good unless allowance is made for the failure of the slab to carry the theoretical proportion of the total longitudinal moment. In other words, after appre-

[19] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 94, Fig. 40*a*.

ciable cracking takes place, the slab acts much less effectively as a longitudina beam than before, even though its effectiveness in carrying loads laterally is relatively unimpaired.

A safe assumption for the beams is that the slab is capable of carrying no part of the static moment. If the computed beam moments are increased arbitrarily so as to account for the total static moment the computed strains are in close agreement with the measured strains. It can be concluded, therefore, that the theoretical analysis can be used as a guide in apportioning the total static moment among the beams for the purpose of design.

d. Concept of Proportion of Wheel Load Carried by Beams.—Although the actual load distribution to the beams is fairly complicated, a fictitious distribution of loading to the beams can be selected to account for the moments in the beams. The maximum moments in the various beams as a result of truck loading are nearly equal, and are approximated closely by assigning part of a wheel load, k, to each beam. This proportion depends on a, b, and H; but within practical limits, for bridges of a given type, the proportion is equal to the spacing of the beams divided by a constant of between 5 ft and 6 ft, depending on whether the stiffness of the beams relative to that of the slab is high or moderate. The proportion of a wheel load "carried" by each beam must be greater than the average obtained from the number of wheel loads on the bridge divided by the number of beams, since different load positions are required to produce maximum moments in various beams.

It is expedient to express k as

$$k = \frac{b}{s} \qquad (3)$$

in which s is some length which depends on the characteristics of the structure.

Live-load moments are reported[12] for standard truck loads on more than 50 I-beam bridges of various span, beam spacing, and relative stiffness of beams to slab. Some of the structures analyzed represented extreme conditions but were investigated to determine trends in the moment coefficients beyond the range of practical applications. Values of s to be used in Eq. 3, in computing maximum moments at midspan of interior beams due to rear wheels of standard trucks, were determined from tabulated moment coefficients.[12] A value that fits practical designs is given by the relation:

$$s = 4.6 \text{ (feet)} + 0.04 \frac{a}{\sqrt{H}} \qquad (4)$$

Similar studies for the effect of rear wheels or of additional trucks in a lane indicate that the same factor k can be used. Consequently, for any truck loading, the proportion of a wheel load to be used in the design will be given by Eqs. 3 and 4.

e. Deflections of Beams.—The longitudinal distribution of loading on the beams is not the same shape for all beams. Therefore, the moments in the beams and the deflections of the beams in general are not proportional, especially for single concentrated loads. This condition is revealed most clearly

in Fig. 18 which shows the moments and deflections at the center of a bridge relative to the maximum values, and to the averages for the five beams, when a single load is placed at the center ($H = 5$ and $b/a = 0.1$). The moments are divided among the beams in a much less uniform manner than the deflections. Consequently, it is necessary to measure strains rather than deflections to obtain an accurate picture of the action of the structure. However, the differences between the patterns of moment and deflection are much smaller for truck loadings than for single concentrations. It is on the safe side but not overly conservative to compute maximum deflections for the same proportion k of a wheel load that is used to determine moments.

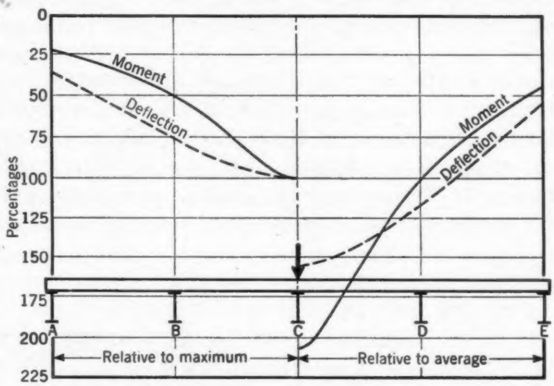

FIG. 18.—RELATIVE MOMENTS AND DEFLECTIONS OF BEAMS FOR A SINGLE LOAD ON THE CENTER LINE OF SPAN ($H = 5$; $b/a = 0.1$)

For more accurate determination of deflections, prepared tables of coefficients[12] may be used. Justification for computing deflections by the theory is apparent from the test results published in 1946, which demonstrate[20] the close agreement between the theoretical values and the test results.

f. Factors Affecting Moments in Slab Due to Concentrated Wheel Loads.—The behavior of the slab may be considered as divided into several component actions, as follows:

(1) The behavior of a single panel acting as a simply-supported rectangular slab. Moments corresponding to such action have been given by Professor Westergaard.[2] (His moments are given for Poisson's ratio = 0.15 instead of zero, as assumed in this paper.)

(2) The effect of continuity with adjacent panels, assuming nondeflecting beams. The maximum moments resulting from wheel loads in action (1) are reduced by continuity, the reduction for representative structures being of the order of 20%.

(3) The effect of the deflections of the supporting beams. The effect of this action is to increase the maximum positive moments, and the increase is of the same general order of magnitude as is the decrease caused by action (2).

The relative magnitudes of these effects, determined from the ordinary theory of flexure of slabs, are shown in Fig. 19 for a particular type of bridge.

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[20] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 97, Fig. 42.

The single-panel moment is determined for Poisson's ratio = 0 and is therefore somewhat less than the value given by Professor Westergaard. The curves in Fig. 19 give influence ordinates for moment at the center of a panel of the slab. The magnitude of the moment at this point, when the load is applied at the point, is determined by a correction to the ordinary theory of flexure of slabs in a manner devised by Professor Westergaard.[21] The effects of continuity on the single-panel moment are shown by the curve for $H = \infty$—that is, for nondeflecting beams. The differences between the curves for $H = \infty$ and for the other values of H are due to deflection of the beams. For a bridge of the proportions indicated, the value of H in a reasonable design is approximately 5. In any case, the greatest part of the maximum positive moment at the center of a panel of the slab, resulting from the combined actions, is the

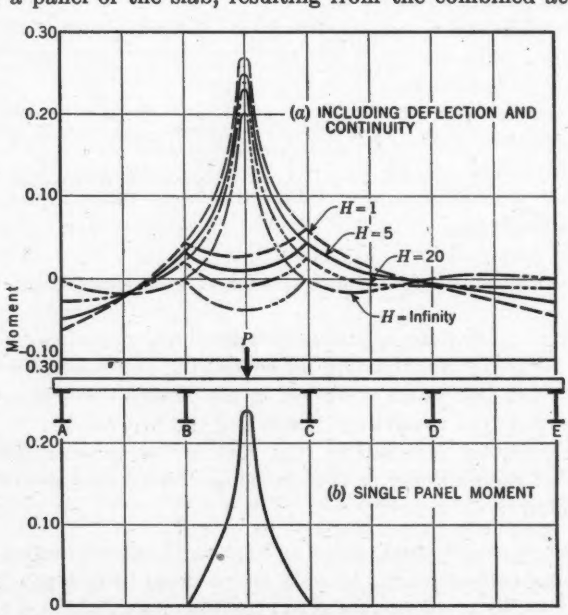

FIG. 19.—INFLUENCE LINES FOR TRANSVERSE MOMENT IN SLAB AT BC AT MIDSPAN (Load on Center Line ($b/a = 0.1$))

moment directly beneath the load point caused by the action of the slab as a single panel.

The theoretical values of the transverse moment and the longitudinal moment (respectively, M_t and M_l) for a single panel of a slab at a point directly beneath a load P are given approximately by the following formulas:

$$M_t = \frac{1.16\,P}{3 + 10\,\frac{c}{b}} \qquad (5a)$$

and

$$M_l = M_t - 0.080\,P \qquad (5b)$$

[21] "Moments in I-Beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1942, pp. 14–15.

In Eqs. 5, c is the diameter of the equivalent circular area over which the wheel load is assumed to be distributed. For H-15 and H-20 loadings the quantity c is ordinarily taken as 15 in. or 1.25 ft.

To keep in mind the relative magnitudes of the moments due to various causes, it is desirable to remember that for ordinary proportions of I-beam bridges M_t may vary between 0.20 P and 0.28 P, whereas M_l may vary between 0.12 P and 0.20 P.

g. Major Design Moments in Slab.—As a result of tests of both single-slab panels and I-beam bridges, it was found that in a reinforced concrete slab the stress in the reinforcing steel immediately beneath a single concentrated load is always considerably less than the stress computed by the theory. This fact is not unreasonable since even the modified theory applying to the immediate region of the load is not properly applicable to reinforced concrete after cracking.

It is impossible to separate into the component actions (1), (2), and (3) the stresses in the slab determined from tests, and it is therefore difficult to state whether the effects of deflection and continuity are indicated reasonably well by the theory. However, test results do indicate good agreement with theoretical beam moments; consequently, it seems reasonable that at least the qualitative values of these effects must be as indicated by the theory.

Typical quarter-scale model bridges were tested in the laboratory. Load-strain curves for slab reinforcement in a number of these bridges, summarized from the test data, demonstrate[13] the discrepancies between the theory and the test results which require some explanation.

A number of factors influence the magnitude of the strains in the steel at the center of a panel of the slab. A major uncertainty is the extent and degree of cracking, for steel stresses are generally low until the concrete cracks. In addition to the direct effect of cracking, there is an indirect effect produced by redistribution of moments on various sections as the relative stiffness of different parts of the structure changes due to cracking—that is, the amount of transverse steel over the beams may have an appreciable influence on the strains in the reinforcement at the center of a panel. If inadequate transverse reinforcement is provided over the beams, the effect of continuity is reduced by cracking of the slab as the result of negative moments over the beams. The result may be an appreciable increase in the positive moments at the centers of the panels. On the other hand, if enough transverse reinforcement is provided over the beams so that the slab does not crack there, an increased part of the moment is attracted to the part of the panel over the beams, and the slab acts to some extent as if it were haunched. The increase in negative moments is accompanied by a decrease in positive moments at the center of the panels.

The effect of torsional resistance of the beams, which is especially marked when there are shear connectors, is to increase the effect of continuity and to reduce the effect of deflections of the beams, resulting in an appreciable net decrease in the positive moments at the centers of the panels.

The test results can be summarized by the statement that the measured steel strains in the reinforcement at the center of a panel of the slab are always considerably less than is indicated by the theory, both for simply-supported

panels and for I-beam bridges, with the exception of very short-span bridges. For the latter, the steel strains after extensive cracking are about equal to the values indicated by the theory.

For short spans, the relative longitudinal stiffness of the slab is fairly high and an appreciable part of the total longitudinal moment is carried by the slab. A rough measure of the proportion of the static moment carried by the slab is the ratio of the total stiffness of the slab to the total stiffness of the beams and the slab on any cross section. With four slab panels and five beams, this ratio, r, becomes

$$r = \frac{4\,b\,E\,I}{4\,b\,E\,I + 5\,E_b\,I_b} = \frac{1}{1 + \frac{5}{4}\frac{a\,H}{b}} \qquad (6)$$

For the short-span bridges tested, the value of r is between 0.23 and 0.07, whereas for the long-span bridges the value of r is 0.02 or less. When the slab cracks in the short-span bridges, the beams must carry a larger part of the total moment; consequently, the slab must distribute laterally a greater part of the load, and in doing so the transverse slab moments are increased. This behavior (at least in part) is an explanation of the fact that the test results indicate measured moments more nearly equal to the theoretical moments for the bridges with large ratios of spacing to span and low values of H.

It is apparent from test results[13] that the slab probably did not crack as the result of negative transverse moments over the beams—at least not until relatively high loads were applied.

For a long-span bridge the effects of front wheels on the moments become of importance, and the effect of deflection of the beams increases relative to the effect of continuity of the panel. Previous design procedures given by the writer have provided for increased design moments for longer spans in agreement with the increase computed from the theory. The nature of the effect of increase in span length on the transverse moment at the center of a panel can be inferred from an examination of typical influence surfaces for transverse moment.[22] As more loads, including the front wheels of the truck with rear wheels at the point under consideration, come on the span, the moment tends to increase because the relative deflections of the beams increase.

The general results of the tests indicate that the theory is considerably on the conservative side. Furthermore, there is a large reserve of strength in the slab beyond the point at which the reinforcement first begins to yield. Final and complete failure does not occur until yielding is general in the entire region near the load. The failure takes place by a kind of punching action in which a cone of concrete is pushed out of the slab. This type of failure occurs at loads of from 50% to 100%, or more, greater than the loads producing first yielding.

Therefore, it appears that a reduction in the theoretical moment in the slab can safely be made for the purposes of design. It is common in floor slabs for buildings, subjected to uniform loads, to reduce the theoretical moments by about 30%. A similar reduction is suggested herein. The design moments

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[22] "Moments in I-beam Bridges," by N. M. Newmark and C. P. Siess, *Bulletin No. 336*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1942, Appendix B.

recommended in Section 7 of this paper are about 70% of the theoretical moments for the longer span bridges, and are slightly higher for short-span bridges—which is in line with the indications from the test results.

h. Secondary Moments in Slab.—Empirical equations giving the approximate magnitude of the maximum negative moment at the center of a panel of the slab, the maximum positive and negative transverse moments in the slab over a beam, and the maximum longitudinal moments in the slab were reported in 1942.[12] At the center of a panel the maximum negative transverse moment is less than one fourth as much as the positive transverse moment, and the proportion decreases with greater flexibility of the beams. Over the beams both positive and negative transverse moments can be produced by different loadings. The negative moments are generally about three fourths as great as the positive moments near the centers of the panels except when the beams are very stiff. The positive moments vary from about 50% of the negative moments when the beams are very flexible to practically nothing for stiff beams.

The relative magnitudes of the secondary transverse moments in the slab, compared with the major design moment, vary at different points along the span of the bridge. For this reason, and because the test results indicate significantly lower stresses than does the theory, the transverse moments to be provided for in a design can be stated most conveniently in terms of a fixed proportion of the steel used in the transverse direction at the centers of the panels. Detailed recommendations are given subsequently in Section 7.

In the region of a concentrated load on a single-panel slab there is a moment in the longitudinal direction about two thirds as great as the moment in the transverse direction. The theoretical longitudinal moment is appreciably increased by the effect of deflection of the beams, and is only slightly reduced by continuity. The theory alone indicates that almost as much longitudinal reinforcement as transverse reinforcement is required for short-span bridges, with somewhat less for long-span bridges. However, both tests and judgment indicate that the structure cannot be seriously harmed if the required theoretical amount of longitudinal steel is not furnished.

An illustration of the effect on the strains in the longitudinal steel caused by varying the amount of longitudinal steel is shown in Fig. 20, where strains are plotted for three quarter-scale models of 20-ft span bridges similar in all respects except in the amount of longitudinal steel. The longitudinal steel for the various bridges was as follows: For WL5b, 1.06%; for W5 and N5, 0.59%; and, for W05, 0.20%. It can be noted that the strains are not increased at all in proportion to the amount of decrease in reinforcing steel. The relationship is typical of secondary "participation" stresses in which the deformations are practically fixed, and the stresses are relatively unchanged by removing or adding material.

However, the tests[13] indicate that the action of the structure is very unsatisfactory if practically no longitudinal steel is furnished, since the action of the slab in distributing loads to the beams is impaired. The cracks in the bridge with only 0.20% of longitudinal reinforcement[23] are practically all in

[23] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 54, Fig. 17.

the transverse direction, and opened up fairly wide, whereas the cracks in the bridge with 1.06% longitudinal reinforcement were more numerous but relatively small and were in a more random pattern. The appearance of the bridge with only nominal longitudinal reinforcement was distinctly unsatisfactory.

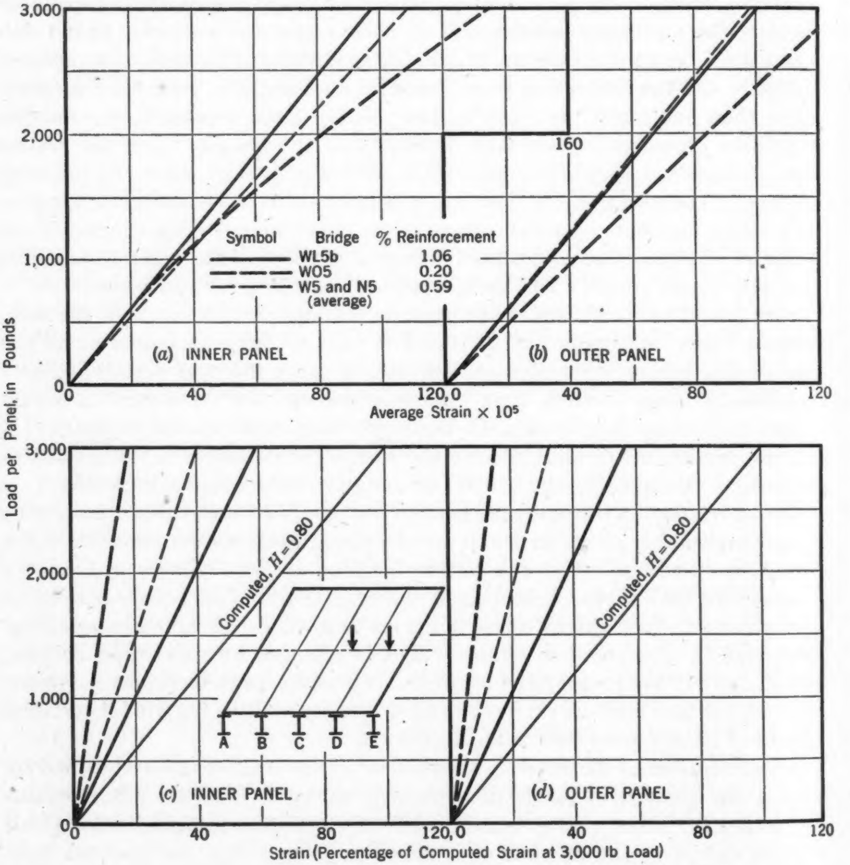

FIG. 20.—LOAD-STRAIN CURVES FOR LONGITUDINAL REINFORCEMENT AT CENTERS OF PANELS; QUARTER-SCALE MODELS OF BRIDGES OF 20-FT SPAN, WITHOUT COMPOSITE ACTION

This comparison illustrates the desirability of furnishing an adequate amount of longitudinal reinforcement. Moreover, the longitudinal steel serves the additional purpose of tying the slab together and resisting to some extent the action of temperature changes and shrinkage.

To prevent disintegration of the concrete from cracking due to other causes than loads, it is desirable to provide a reasonable amount of longitudinal reinforcement. In structures where longitudinal continuity of the slab is necessary for the safety of the structure, as in a composite I-beam bridge, an amount of longitudinal steel is desirable sufficient to prevent shrinkage cracks from open-

ing up and impairing the safety of the structure—in spite of the fact that the analysis indicates much lower requirements in the amount of reinforcement than those for noncomposite bridges.

3. Distribution of Dead Load to Slab and Stringers in Simple-Span Right I-Beam Bridges

a. Moments in Beams.—The dead-load moments in the beams may be computed by dividing the total dead-load moment at the center of the bridge equally among the beams. In general, this procedure is the same as considering each beam to carry its own weight, the weight of one panel of the slab, and whatever paving allowance is considered. The added weight of curbs, sidewalk, and handrails will compensate for the fact that only a half panel of the slab is carried by an edge beam. Even when the edge beam is more heavily loaded than the interior beams, the action of the slab transmits a part of the load to the interior beams, and all the beams share to some extent in carrying the additional load applied to the edge beam. The relative proportions of dead-load and live-load moments in the beams may vary over a considerable range depending primarily on the slab thickness and on the span of the bridge. For designs made according to the provisions recommended herein, dead-load and live-load moments are approximately equal for spans of about 65 ft; dead-load moments exceed live-load moments by some 20% for spans of about 85 ft; and live-load moments exceed dead-load moments by perhaps 50% for spans of about 45 ft. However, an increase in slab thickness of 1 in. in 6½ in. will increase the dead-load moment about 15%. Therefore, the design of the beam is fairly sensitive to changes in slab thickness and to provisions for wearing surface or added pavement.

b. Moments in Slab.—Dead-load moments in the slab are affected by the manner of construction and by the sequence of casting the concrete.[12] Therefore, they are subject to considerable uncertainty. Fortunately, the dead-load moments in the slab are small—ordinarily only about 10% of the live-load moments.

In a bridge constructed without shoring, the weight of the slab is transferred to the beams by the formwork immediately after casting. When the formwork is removed, the reactions of the slab on the beams are changed from simple-span reactions to continuous beam reactions. The consequent deflections of the beams change the relative proportions of positive and negative moments in the slab. In general, the positive moments are increased and the negative moments reduced from the continuous beam moments. This change is greater the more flexible the beams.

When a uniform load is placed over the entire structure, as an added pavement applied subsequent to removal of the forms, large positive moments are introduced over the entire structure. On the other hand, a loading applied to the outer beams, such as a curb or handrail cast after the roadway forms are removed, produces appreciable negative moments in the transverse direction over the entire structure. For these reasons it is felt that, unless the construction sequence is controlled, provision should be made in the design for a positive dead-load moment in the transverse direction at the centers of panels of the

slab of magnitude $\frac{1}{8} w b^2$, in which w is the intensity of the dead load per unit of area. When the beams are unusually stiff, or where composite construction is used, this value may be reduced.

4. Composite I-Beam Bridges

a. Advantages of Composite Action.—Composite action of the slab with the beams may exist to a limited extent, or even almost completely, although no provision for such action is made in the design. On the other hand, it is impossible to depend on composite action unless positive provision is made for adequate shear connectors, since temperature changes, shrinkage, and the effects of wheel loads may sooner or later destroy the adhesive bond between the slab and the beams.

In a composite I-beam bridge a panel of the slab acts as the upper flange of the composite beam. Tests indicate that, for usual proportions, the full width of the slab is effective. The moment of inertia of the composite section may be from two to three times as great as that for a design that does not utilize composite action. Therefore, the composite bridge has a greater value of H and the beams have relatively greater maximum moments compared with the average. Consequently, in the design of a composite I-beam bridge, a somewhat greater proportion of a wheel load must be assumed to be carried by the beams than in a noncomposite bridge.

The distance to the bottom flange from the neutral axis of the composite section is considerably greater than in a noncomposite design. Therefore, the stress controlling the design of a composite beam is decreased much less proportionately than the moment of inertia is increased. In general, there will be a reduction in depth and a slight saving in weight of the beams if rolled sections are used as the beams in the composite construction; a greater saving in weight will result from the use of cover plates on the bottom flanges; and a considerable saving is obtained by use of a section with a heavier bottom flange than top flange.

The increased strength and stiffness of a composite beam compared with those of an I-beam acting alone are obtained without impairing the ability of the slab to perform its primary function in distributing the loads laterally to the beams as a roadway slab. This conclusion was indicated by the theory and was shown by the tests.

In the negative moment regions of continuous bridges the advantages of composite action are not so marked. Indeed, it is questionable whether there is a disadvantage in having the slab subjected to appreciable tensile stresses as the upper flange of a composite beam. However, even though it may not be desired in continuous bridges, there may be partial or complete composite action because of the bond between the concrete slab and the steel beams. Although this bond cannot be depended on in design, it may exist as a disturbing and complicating element.

b. Shear Connectors.—Shear connectors to insure composite action may be composed of channels, plates, angles, or other structural elements, welded or riveted to the top flange of the beam, and embedded in the concrete of the slab. To prevent uplift there should be a mechanical anchorage of the connector in

the concrete. The load to be carried by the shear connector may be computed by the ordinary formulas of elementary mechanics from the shear on the composite T-beam section.

When the composite structure is constructed in the usual manner, without falsework supporting the beams during construction of the slab, the dead load from both the slab and the beam is carried by the steel beam section alone, but the live load is carried by the composite T-beam section. Where the dead-load shear is carried by the steel beam section alone, it should be noted that the shear connectors do not have to provide for the dead-load action.

The dead-load shear varies linearly from a value equal to the reaction at the end of a beam to zero at the center. It is recommended that the live-load shear in a beam be computed for either truck or truck train loadings by the use of the same proportion, k, of a wheel load that is used in determining the moment in the beam. The total force acting on any shear connector may then be computed from the longitudinal shear per unit of length between the beam and the slab, multiplied by the spacing between the shear connectors.

When the shear connector is capable of resisting forces tending to separate the slab from the beam, a greatly increased effectiveness of the shear connector is obtained. The tying-down action of the connector serves to make the natural adhesive bond between the concrete slab and the steel beam effective also in resisting shearing forces.

For design purposes, the tentative recommendation is made that the distribution of load over the face of the shear connector be considered linear, from zero at the top of the connector to a maximum at the junction of the connector and the top flange of the beam. The maximum stress in bearing on the concrete may be taken equal to the usual compressive working stress for concrete, and the maximum flexural stress in the shear connector may be taken as the usual working stress for the steel. The spacing of the shear connectors should be not more than from three to four times the depth of the slab. The usual criteria should cover the design of the connection between the shear connector and the beam.

The model tests and small-scale tests on composite beams indicate that, when the shear connector is a channel having one flange attached to the beam, the foregoing working stresses may safely be doubled, or the load that must be provided for by the shear connector may be reduced by one half. Tests of quarter-scale models of I-beam bridges with such shear connectors indicated that the strength of the slab and of the beams may be developed without any evidence of failure of the shear connectors, even with a less conservative design than that proposed.

In the bridges tested, no evidence of inadequate action of shear connectors was detected even at loads causing failure of the slab and beams. Corresponding to the shear connectors used in several of the structures, the prototype shear connectors in a full-sized bridge would be 6-in. lengths of 4-in. channels with a web thickness of $\frac{1}{2}$ in., spaced 25 in. apart.

Somewhat less conservative recommendations are under consideration and the entire problem is the subject of an extensive series of tests which are described more fully in Professor Siess' paper in this Symposium.

c. Effect of Shoring Beams During Construction.—For bridges constructed without falsework, the dead-load stresses in the bottom flanges of the beams are a fairly large proportion of the total stress. The dead-load stresses can be reduced by use of temporary shores during construction so as to permit the dead load to be carried by the composite section instead of by the steel beams alone. By jacking up the beams even more, a favorable pre-stress in the beams can be introduced.

Studies indicate that the use of from one to three shores under each beam does not produce quite the same effect as a continuous rigid support. When the shores are released, the maximum effective dead-load moment is slightly different from the value corresponding to release of a continuous support. However, for practical purposes the shoring used may be considered as fully effective. Then the dead-load and live-load stresses will be approximately proportional to the magnitudes of the dead-load and the live-load moments.

In a design made for the usual working stresses for a loading consisting of one dead load plus one live load applied to the structure, the effect of shoring in composite design is to permit the use of a lighter beam than that corresponding to composite design without shores.

The decrease in weight, in a series of representative designs made according to the procedures described herein, amounted to about 15% for rolled beams, and ranged from 15% to 30% for welded built-up sections.

However, it is believed that these data are misleading since the resulting structure has a reduced ultimate strength and a reduced usable maximum capacity compared with designs in which shoring is not contemplated. This conclusion is entirely aside from any considerations of economy of construction, although the major reasons for the popularity of I-beam bridges are the ease and simplicity of the construction procedure.

d. Capacity of Composite I-*Beam Bridges.*—Properly designed shear connectors will develop the ultimate strength of the slab and beams before showing any evidences of failure to function properly. Weaker shear connectors may also develop almost the same ultimate strength of the structure, but may show considerable slip in doing so; and at maximum loads the structure will have considerably larger deflections. However, extremely weak shear connectors may fail at loads such that the noncomposite structure resulting from the failure of the shear connectors will not have sufficient resistance to carry the load on the bridge. Such a failure can be catastrophic, and is not to be tolerated; consequently, it is reasonable to be extremely conservative in the design of shear connectors.

In general, the dead-load stresses and deflections are larger, and the live-load stresses and deflections are smaller, for a composite bridge than for a noncomposite bridge of the same span length. Then the capacity, measured in number of live loads, will usually be greater for the composite bridge. To take advantage of the increased strength of the composite structure, not only must the shear connectors be conservatively designed, but the concrete slab must be adequately reinforced in the longitudinal direction to prevent serious opening of shrinkage cracks. Where the continuity of the slab is interrupted too greatly the upper flange of the composite section is materially weakened.

Consequently, although live-load stresses in the slab do not indicate a need for very much longitudinal steel in the slab, sufficient amounts of such steel must nevertheless be provided. It does not seem desirable to use an amount of steel less than about 0.5% of the effective concrete area, to provide for temperature and shrinkage effects.

The results of the tests appear to indicate that the action of bridges with composite beams agrees with the theoretical analysis in about the same way as does the action of bridges with noncomposite beams. The effect of composite action is to increase appreciably the strength of the roadway slab. There appears to be a reduction in the transverse stresses in the reinforcement, possibly due to the increased torsional resistance of the composite I-beams. There is also an increase in the loads required to produce failure of the slab by punching.

5. Skew I-Beam Bridges

a. General Considerations.—No analyses of skew I-beam bridges are available, but laboratory tests have been made on five skew bridges having angles of skew of 30° and 60°. In all respects, except for angle of skew, the bridges were similar to corresponding right I-beam bridges previously tested. The results of the test indicated that for a 30° angle of skew the action of the skew bridge was not greatly different from the action of a corresponding right bridge. However, for the 60° angle of skew, there was a considerable difference, with appreciably greater relative deflections of the structure at a given cross section and with markedly higher strains in the slab reinforcement.

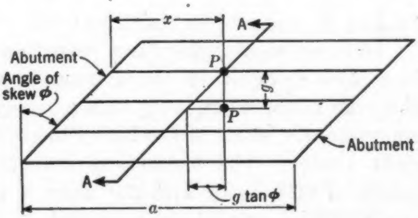

Fig. 21.—Skew Section Through a Skew Bridge, Showing the Location of Rear Wheel Loads

Consider the plan of a skew bridge shown in Fig. 21 with a section AA parallel to the abutments. For any loading the total bending moment normal to the section can be computed by the laws of statics. For the loading shown in Fig. 21, with one wheel of a truck at the section considered, the total moment M_n normal to the section is given by the expression:

$$M_n = 2\,P\,x \cos\phi\left(1 - \frac{x}{a} - \frac{g}{2\,a}\tan\phi\right) \qquad (7a)$$

in which ϕ is the angle of skew; g is the spacing of the wheels on the axle; a is the span of I-beams; and x is the distance in the direction of the span from a support to the section. In addition, there may be a twisting moment in the plane of the section, but the twisting moment is not statically determinate.

b. Effect of Skew on Moments in Beams.—An approximate value of the sum of the bending moments in the beams can be derived by making the assumptions that the slab carries only a negligible part of the total moment on the section and that the twisting moment in each beam on the section normal to

the axis of the beam is negligible. With this procedure the sum of the moments in the beams, ΣM, on section AA in Fig. 21, is as follows:

$$\Sigma M = 2\,P\,x\left(1 - \frac{x}{a} - \frac{g}{2\,a}\tan\phi\right) \qquad (7b)$$

By inserting the proper values in Eq. 7*b*, the moments on the skew center line (or the maximum value of ΣM which is slightly greater than the value for the skew center line) can be obtained. For a right bridge, $\phi = 0$ and the maximum value is the same as that on the center line—namely $\frac{1}{2}\,P\,a$. The values of the average beam moments in the skew bridge, M_s, relative to those in a right bridge, M_r, are therefore given by the following formulas: For skew center line, $x = \frac{a}{2}$—

$$\frac{M_s}{M_r} = 1 - \frac{g}{a}\tan\phi \qquad (8a)$$

and, for maximum ratio, M_s/M_r—

$$\frac{M_s}{M_r} = \left(1 - \frac{g}{2\,a}\tan\phi\right)^2 \qquad (8b)$$

In Eqs. 8, g should be taken as 6 ft.

In other words, there is a reduction in the average moments in the beams in a skew bridge from the average in a right bridge because of the fact that the axles of the truck loads on the bridge are not parallel to the abutment; and, consequently, some of the heavy loads are closer to the abutments than in the right bridge. The theoretical reduction in average beam moments for a bridge of 60-ft span with 30° skew is about 6%, and for one with 60° skew about 17%.

The reduction in average moment on a section of a skew bridge does not necessarily mean that there is a reduction in moment in all the individual beams at a section. The test results indicate a more nonuniform distribution of strain in the beams at a given section than in a right bridge. However, some reduction in strain in the beams for bridges with large angles of skew is indicated, but practically no reduction is indicated for bridges with 30° skew.

Load-strain curves for the beams of one of the 60° skew bridges are shown in Fig. 22. The ratio of the measured maximum beam strains for the skew bridge to those in a corresponding right bridge is about 0.80. However, for the loading considered, the average moment on the skew section where the average is greatest is only about 71% of the moment in a right bridge for the same type of loading. Although there is some indication from the tests that shifting the loads to produce a greater moment on a skew section would not greatly increase the maximum measured strains, such a test unfortunately was not actually performed. Consequently, to be conservative, a possible increase must be inferred in the ratio of maximum measured strains from 0.80 to 0.94 if the average moment is increased from 71% to 83% of the corresponding moment in a right bridge. This argument suggests that the reduction in maximum moments in a skew bridge is perhaps about half as much as the reduction in

average moment on a skew section; but, in view of the uncertainties involved, it is recommended that the beams of skew bridges be designed for the same proportion of a wheel load that applies in the case of a right bridge of the same beam span.

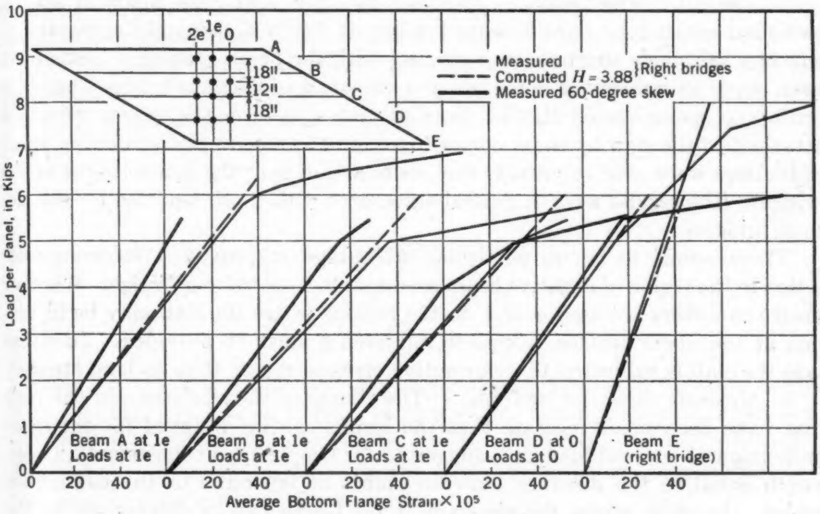

FIG. 22.—LOAD-STRAIN CURVES FOR BEAMS; QUARTER-SCALE MODELS OF BRIDGES OF 60-FT SPAN, WITHOUT COMPOSITE ACTION

c. Effect of Skew on Moments in Slab.—It is difficult to generalize about the moments in the slab in skew bridges from the results of the tests. First yielding of the reinforcement occurred at lower loads for the skew bridges and the loads were smaller with greater angles of skew. However, the ultimate capacity of the slab, as measured by the loads producing punching failures, appeared to be independent of the angle of skew—at least for skews up to 60°. A small but significant increase in strain in the transverse reinforcement for skew bridges over corresponding right bridges was observed in the tests.

The results of the skew bridge tests indicated increased strains in the longitudinal as well as in the transverse reinforcement in the slab. The increase arises from the more nonuniform deflections of the beams in a skew bridge. However, there is an additional reason for the increased strain in the longitudinal steel. The slab tends to carry the load directly to the abutments by the shortest route, which is at an angle to the beams; consequently, the principal moments in the slab tend to deviate from the directions of the beams. For this reason it is unwise to reduce the amount of longitudinal steel in the slab of skew bridges. To do so might invite cracking of a nature and magnitude that would be undesirable.

In view of the fact that the design recommendations for slabs are conservative in any case and furthermore since the ultimate strength of the slab does not seem to be affected by skew, it is tentatively suggested that no change in

the provisions for the design of the slab be made in skew bridges, except to emphasize the requirement of longitudinal reinforcement.

6. Continuous I-Beam Bridges

a. General Considerations.—Although analyses have been made of several two-span continuous right I-beam bridges, it was unfortunately necessary to consider relatively short spans compared with the beam spacing. Tests have been made on quarter-scale models of a two-span continuous bridge where the prototype spans are 60 ft. In the first test structure, the beams were not attached to the slab by shear connectors. In the second test structure, shear connectors were used to provide composite action over the entire length of the bridge. The results were in general agreement with those reported for simple-span bridges.

There seems to be no particular advantage in providing for composite action in the region of negative beam moments in a continuous bridge. Whether shear connectors are used or not, in the regions where the slab may be in tension at the upper surface, adequate reinforcing must be provided. In either case the slab is subjected to deformation stresses rather than to load stresses.

b. Moments in Beams and Slab.—The results of the analyses and the tests that have been made indicate that the beams can be designed for the same proportion of a wheel load as is suggested for simple-span bridges with a span length equal to the distance between points of inflection of the continuous bridge. In other words, the moments in the beams can be determined by the usual procedure for a continuous beam subjected to a particular proportion of a wheel load and to the proper dead load.

The indications are also that the slab may be designed as for a simple-span bridge with a span of the beams, a, equal to the distance between the "points of inflection" for the particular span of the continuous bridge.

In the central part of the spans of the continuous bridge, the secondary moments in the slab are approximately the same as those in a simple-span bridge. However, over the supports, the secondary moments are practically the same as those in a bridge having infinitely rigid or nondeflecting beams. The major change is in the negative transverse moments over the beams. These are likely to be somewhat larger over the supports than in the central parts of the spans. Nevertheless, the model structures that have been tested indicate that it is not necessary to have as much steel provided for negative moment over the beams as for positive moment at the centers of the panels. The test structures had only two thirds as much transverse steel at the top of the slab as at the bottom, but no evidence of serious weakness or of incipient failure was observed. The situation might be worse in a skew-span continuous bridge, however, and it seems desirable to use more steel over the beams in the regions of negative beam moments, than in the regions of positive beam moments.

7. Design Recommendations for Slab

a. Major Design Moment.—The test results that have been cited indicate that the maximum positive transverse moment in the slab, as given by the

theory, corresponds to considerably larger strains than have been measured in the tests except for a few instances. Moreover, there is a factor of safety beyond the first yielding of slab reinforcement of at least 50% before punching failure occurs. Consequently, it appears permissible to design the slab for a moment smaller than the theoretical moment. A reduction of about 30% appears to be reasonable, and is the basis of the following recommendations.

The maximum positive live-load transverse moment in the central part of panels of the slab, expressed in foot-pounds, for a 1-ft width, is to be computed from the following formulas: For spans of beams less than 60 ft—

$$M_t = \frac{P_m}{3 + 10\,\frac{c}{b}} \qquad (9a)$$

and, for spans of beams greater than 60 ft—

$$M_t \doteq \frac{P_m}{3 + 10\,\frac{c}{b}} + \frac{P_m\,(a - 60)}{1{,}000} \qquad (9b)$$

In continuous spans, a is to be taken as the distance between points of contraflexure in the region considered. In Eqs. 9, P_m is the maximum wheel load, including impact, in pounds; c is the width of loaded area over which P is distributed, ordinarily 1.25 ft; b is the center-to-center spacing of beams, in feet; and a is the span of beams, in feet.

The positive transverse dead-load moment M_w is to be taken as

$$M_w = \tfrac{1}{8}\,w\,b^2 \qquad (10)$$

in which w is the intensity of uniform dead load of slab and paving surface, in pounds per square foot. In bridges in which composite action is provided for by shear connectors, or in concrete T-beam bridges, the moments given by Eqs. 9 and 10 may be reduced 10%.

TABLE 1.—Relative Amounts of Reinforcement Required in a Slab

Place	Transverse		Longitudinal	Notes
	Center of panel	Over the beams	Over the entire panel	[a] This value applies to simple-span bridges and to the positive moment regions of continuous bridges. In the negative moment regions the value should be 0.8. [b] For composite bridges, and for all skew bridges, the reinforcement should be not less than 0.5% of the gross concrete area.
Bottom	1.0	0.4	0.5[b]	
Top	0.2	0.7[a]	0.2	

b. Arrangement of Reinforcement.—The areas of steel per unit of width provided at other points and in various directions in the slab shall be at least as great as those given in Table 1 in terms of the area of steel per unit of width in the transverse direction at the bottom of the central parts of panels. Where bottom bars are bent up to provide reinforcement over the beams, the points of bend at the bottom should not be greater than 0.25 b from the center of the beams.

8. Design Recommendations for Beams

a. Dependence of Moments in Beams on Depth of Slab.—The design of the beams of an I-beam bridge is dependent on the depth of the slab in two respects: (1) The depth of the slab affects the relative stiffness of the beams and, consequently, the magnitude of the live-load moment in the beams; and (2) the depth of the slab influences the dead load on the beams and thereby the magnitude of the dead-load moment in the beams. For example, in a bridge of 65-ft span and 6.75-ft beam spacing, a 7.25-in. slab (which is approximately that required for an H-20 single wheel concentration of 16 kips plus impact) gives a dead-load moment in the beams of 420 ft-kips, whereas a 6.25-in. slab, which is approximately that required for an H-15 single wheel concentration of 12 kips plus impact, gives a dead-load moment of 368 ft-kips. If the beams in these bridges are to be designed for H-20 truck loading, the proportion of the wheel load assigned to each beam is less for the bridge in which the slab is thicker. The values recommended in Table 2 give (for the live-load moments

TABLE 2.—Values of s (in Feet) To Be Used in the Equation $k = b/s$

Type of structure	Noncomposite I-beam bridges	Composite I-beam bridges
Bridges designed for a standard H-truck loading in which the slab is designed for a single wheel concentration of 0.4 times the total weight of the truck plus impact	5.7	5.3
Bridges designed for an H-20 truck loading in which the slab is designed for a single wheel concentration of only 12,000 lb plus impact[a]	5.5	5.2

[a] The values given in the first line may be used if the depth of the slab is made as great as would be required for a 16,000-lb wheel concentration, plus impact. This provision is made to insure that the slab will have adequate stiffness to distribute the loads to the beams, in order to take advantage of the reduced live-load moments.

in the two bridges) the quantities 471 ft-kips and 488 ft-kips, respectively, with consequent total moments of 891 ft-kips in the structure with the thicker slab and 856 ft-kips in the structure with the thinner slab. The structure in which the slab is designed for a heavier wheel load requires a heavier beam. It appears that any loss in less effective distribution by the thinner slab is more than regained by the material reduction in dead load.

It is worth mention, in this regard, that, if the design is based on a so-called "limit load" producing yielding of the beams, with an appropriate factor of safety, there will be less difference in the two structures. For example, a design based on dead load plus 2.5 live loads at a stress of 33,000 lb per sq in. would require a 36-in., 170-lb wide-flange beam in either case. It is easily noted from the values quoted that the structure with the heavier slab has a somewhat greater live-load rating than does the structure with the thinner slab.

The design coefficients discussed herein are developed for the working stresses specified in the AASHO specifications—namely, 18,000 lb per sq in. for structural steel and reinforcing bars, and 1,000 lb per sq in. for concrete. The modulus of elasticity of concrete is assumed to be 3,000,000 lb per sq in. for live load, and 1,000,000 lb per sq in. for sustained loads, to include the

effect of plastic flow. Other working stresses may affect slightly the ranges in values of relative stiffness of the beams, H, and may require slight changes in Table 2. However, Eqs. 2, 3, and 4 would remain valid for determining beam moments for any case considered.

b. Dead-Load and Live-Load Moments in Beams.—The provisions herein apply only to bridges with four or more parallel beams of the same size, with uniform spacings between beams of 5 ft to 8 ft. A wheel load may pass over an edge beam, but may not extend outside the edge beams. The live-load moments in all beams are to be computed for a proportion, k, of a wheel load, according to Eq. 3—namely, $k = \frac{b}{s}$, in which b is the center-to-center spacing of the beams, in feet, and s may be computed from Eq. 4 or taken from Table 2. However, in no case is k to be less than the number of wheel loads that may come on any cross section of the bridge, divided by the total number of beams.

The dead-load moments in all beams are to be computed by dividing the total dead load of the structure equally among the beams; but the dead load apportioned to any beam should not be less than the weight of the beam, the weight of one panel of the slab, and the weight of any wearing surface over a width of one panel of the slab.

The same factors should be used for continuous bridges, and skew bridges as for simple-span bridges.

c. Composite I-*Beams.*—For composite design, the width of the slab that may be considered as effective as the upper flange of the composite beam may be taken as the panel width. The modular ratio, n, should be taken as 10 for live loads and for loads of short-time duration; and as 30 for dead loads or for long-time loads, such as the weight of added pavement or wearing surface.

The local longitudinal flexural stress in compression at the top of the slab under the wheel load, to be added to the compressive stress in the concrete acting as a part of the composite beam section, should be 30% of the transverse compressive stress in the slab at the same point. The total compressive stress should be limited to the value of the usual working stress for concrete in compression. The foregoing provisions may be applied without modification to concrete T-beam bridges. However, for such bridges, the value of s to be used in Eq. 3 should be 5.0 ft.

9. Practical Significance of Design Recommendations

The moments recommended herein for design of the slab differ somewhat from the provisions of the 1944 AASHO specifications. The value of live-load moment given by Eq. 9*a* is numerically the same as that in the specifications except for the use of a center-to-center span in the equation. The difference is slight and makes the equation easier to use since it is not necessary to design the beams to find the flange width before computing the slab moments. The dead-load moment given by Eq. 10 is greater than would be computed from the AASHO specifications. The reason for the increased dead-load moment is explained in Section 3. The provision for increase in slab moments for spans longer than 60 ft, in Eq. 9*b*, differs also from the specifications, and is included to take account of the increase in theoretical moments for longer spans arising

from an increased effect of deflection of the beams and from the effect of front wheels.

The negative moments over the beams provided for in this paper are considerably less than those provided for by the specifications, but they are consistent with the results of analyses and tests. The procedure outlined herein offers a more rational guide to the proper reinforcement of the slab of an I-beam bridge than do previous studies. The numerical values, based on the results of tests as well as analyses, are somewhat less conservative than the values previously presented by the writer. The design moments in the beams are less conservative than those given by the specifications, but the results of analyses and tests justify the departure from previous standards.

In this regard, it must be emphasized that the proportion of a wheel load, k, used in the design of the beams is a function of the relative stiffness of the beams to the slab and consequently of the thickness of the slab. Where changes in specifications and working stresses, or other causes, lead to decreased slab thickness, the values of k must increase.

The provisions for the design of the slab and the beams, although less conservative than present practice in some respects, are justified by the extensive analyses and tests that have been made on this type of structure. The recommendations have been tried on actual designs, and comparisons have been made with standard practice in many cases. It is felt that the provisions given herein are not only reasonable, but practicable, and should lead to more efficient utilization of steel and concrete in I-beam highway bridges.

COMPOSITE CONSTRUCTION FOR I-BEAM BRIDGES

By C. P. Siess,[24] Assoc. M. ASCE

Synopsis

The results of analytical and experimental studies of composite construction for I-beam highway bridges are presented in this paper. Comparative design studies were made to determine the savings in weight possible from the use of the following types of composite beams: Rolled wide-flange beams, rolled beams with welded cover plates, welded built-up beams, beams consisting of two structural T-sections welded together, and rolled unsymmetrical beams. The savings in weight as compared to noncomposite construction ranged from 8% for the ordinary rolled beams to 30% or more for welded built-up beams.

A number of problems encountered in the design of composite beams are discussed. Included are the effect of slip between slab and beams, the distribution of compressive stress across the width of the slab, and questions regarding the proper value of the modular ratio n to be used in design. Empirical formulas for use in proportioning composite beams are also given.

Tests of shear connectors, particularly of the channel type, are described. The effects of the following variables were investigated: Width of connector, thickness of channel web, and compressive strength of the concrete in the slab. Certain of the results obtained in these tests may be explained by the concept of dowel-like action of flexible shear connectors. According to this concept, the nature of the distribution of bearing pressure on the connector is a function of the relative stiffness of the connector and the concrete.

Introduction

The several problems associated with the design of composite I-beam highway bridges which are discussed in this paper may be divided into three groups:

(1) Comparative design studies of I-beam bridges to determine the savings in weight possible from the use of various types of composite beams;

(2) Experimental and analytical studies of the behavior of composite beams and composite bridges; and

(3) Studies of the behavior of shear connectors for use in composite I-beam bridges.

Although composite construction is not, in itself, a new development, its application to highway bridges is of comparatively recent origin. The I-beam bridge is almost ideally suited to the effective use of composite construction, and the number of such structures has shown a marked increase. An ordinary

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I-beam bridge consists essentially of a reinforced concrete floor slab supported on a number of parallel steel I-beam stringers extending in the direction of traffic. In a composite I-beam bridge, some type of shear connection is provided to transmit horizontal shear between the slab and the beams. The longitudinal load-carrying member is thus transformed from a simple steel I-beam to a composite steel and concrete T-beam in which the concrete floor slab acts as the top flange. In this type of construction the slab serves two functions: First, it transmits load laterally to the beams and, second, it assists the steel stringer in carrying the load longitudinally to the piers. In its first function the slab acts in bending; in its second, it acts chiefly in compression. Tests have demonstrated that the capacity of the slab to perform its primary function of acting as a roadway deck is not impaired by its action as part of the composite beam.

PART A. SAVINGS IN WEIGHT WITH COMPOSITE CONSTRUCTION

1. General Considerations

Both the strength and the stiffness of a composite beam are appreciably greater than the corresponding properties of the steel beam acting alone. The increase in strength resulting from the use of composite construction permits the use of a lighter beam section than would otherwise be required, and, in many instances, permits the use of a shallower beam. Moreover, even with a lighter steel section, the stiffness of the composite beam will be from two to three times as great as the stiffness of the original noncomposite beam. In most cases, the increased stiffness is effective only for live load, since bridges of this type are commonly constructed without shoring beneath the beams, and dead load is carried by the steel beam acting alone. Since the size of the steel beam may be decreased in composite construction, there is likely to be some increase in dead-load deflection for bridges of this type. However, since dead-load deflection can easily be provided for by cambering the beams, this increase is probably not significant. Of greater importance is the considerable decrease in live-load deflection resulting from the increased stiffness of the composite beam. Although present knowledge concerning impact on highway bridges is too meager to permit any definite statements, it seems reasonable to believe that from the standpoint of impact and vibration the stiffer structure is to be desired.

2. Outline of Studies of Weight Savings

Studies have been made of the savings in weight resulting from the use of composite construction utilizing several different types of beams. Comparative designs have been made for simple-span I-beam bridges having span lengths of 30 ft to 90 ft and beam spacings of 5 ft to 7 ft, and designed for both H-15 and H-20 loadings.

Both analytical studies and laboratory tests have indicated that the beams in a composite bridge should be designed for a greater live-load moment than those in a noncomposite structure. The reason for this is that, because

of the greater stiffness of the beams relative to that of the slab, there is less lateral distribution of the wheel-load concentrations. In a previous publication[16] it was recommended that the proportion of a wheel load used in the design of a single beam be taken as $\frac{b}{6}$ and $\frac{b}{5.5}$, respectively, for noncomposite and composite structures. The quantity b is the spacing of the beams, in feet. This recommendation, providing for a 9% greater moment in the beams of the composite bridges, was followed in the designs reported herein. Professor Newmark has proposed a modification of the foregoing recommendation in which the terms $\frac{b}{5.7}$ and $\frac{b}{5.3}$ are substituted for the previous values. For these latter coefficients, the moment for the composite beams is only 7.5% greater than that for the noncomposite beams. This discrepancy between the specifications used and those recommended by Professor Newmark should have no appreciable effect on the comparisons.

Except for the designs given subsequently in Section 6, it was assumed in all cases that dead-load moments are resisted by the steel beam acting alone, and that only live-load moments are carried by the composite section. The results of the various studies are summarized in Table 3, in which the relative

TABLE 3.—RELATIVE WEIGHTS OF STEEL BEAMS IN COMPOSITE I-BEAM BRIDGES

Non-composite rolled beams	Composite Beams							
	Symmetrical Rolled Beams				Unsymmetrical rolled beam, without temporary supports	Double T-section without temporary supports	Welded Beams[b]	
	Without Cover Plate		With Cover Plate[a]				Without temporary supports	With temporary supports
	Without temporary supports	With temporary supports	Without temporary supports	With temporary supports				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
100	92	77	76	64	82	82	69	40 to 60
	100	84	83	70	89	89	74	45 to 70
		100	99	83	106	106	88	53 to 75

[a] Average weight per foot based on cover plate 0.6 of the length of beam. [b] Maximum weight per foot.

weights of the steel beams are compared for the various types of structures considered. In making these comparisons, an amount equal to the weight of the shear connectors has been added to the weights of the composite beams. Three sets of relative values are given to facilitate comparisons of rolled beams, both composite and noncomposite, with the other types of beams considered.

3. Rolled Wide-Flange Beams in Composite Bridges

The average decrease in weight of the beams resulting from the use of composite construction with rolled beams was 8%, as indicated in Col. 2, Table 3. In half of the designs made, the depth of the beams could be reduced by 3 in.

If the most economical depth of beam cannot be used because of clearance requirements, the savings in weight arising from the use of composite construction may be more than twice as great as those quoted, and the shallower depth need not be accompanied by increased flexibility of the bridge in so far as live loads are concerned.

4. Unsymmetrical Beams for Composite Bridges

Because the slab acts as a very heavy cover plate, the neutral axis of a composite beam is raised some distance above mid-depth of the steel beam, and the stresses in the top flange of a symmetrical beam are appreciably smaller than those in the bottom flange. This condition suggests that greater reductions in weight might be obtained with the use of beams having larger bottom flanges than top flanges.

The economies permitted by the use of unsymmetrical sections have been investigated by a limited number of comparative designs for three types of beams: (*a*) Rolled wide-flange beams with cover plates welded to the bottom flanges; (*b*) built-up welded beams; and (*c*) beams consisting of two structural **T**-sections welded together.

(*a*) *Rolled Beams with Cover Plates.*—As indicated by the values in Col. 4, Table 3, the use of rolled wide-flange beams with cover plates in composite bridges was found to result in an average saving in weight of 24% as compared to bridges without composite action. Furthermore, the depths of the composite beams were from 3 in. to 6 in. less, except in those cases where the heavy series of 36-in. beams were required in the noncomposite design.

(*b*) *Built-Up Welded Beams.*—A more efficient section from the standpoint of weight reduction may be obtained by welding together two flange plates and a web plate to form an unsymmetrical **I**-beam. In making comparative designs for economy studies of this type of beam, two conditions were imposed which have an appreciable effect on the results obtained: (1) The webs were made not less than 1/87 times the clear depth, a condition which it is believed provides adequate safety against buckling and obviates the need for stiffeners; and (2) a minimum top flange section, 6 in. by $\frac{3}{8}$ in., was specified to provide space for the attachment of shear connectors.

At first the proportioning of these welded beams was a cut-and-try operation since the depth of the beam as well as the areas of both flanges could be varied. However, as a result of these studies, it was found that the economical depth of beam was very nearly equal to the depth of the rolled wide-flange section required in a composite design. Furthermore, after a great many designs had been made, a simple semi-empirical procedure was developed for proportioning the flanges in such a manner that the total dead-load plus live-load stresses would be equal in the two flanges.

The results of these studies are summarized in Col. 8, Table 3. It was found that the weight of welded beams was on the average about 10% less than that of composite rolled beams with cover plates, 26% less than that of composite rolled beams without cover plates, and 31% less than that of noncomposite rolled beams. If the flanges of the welded beams are made smaller in regions of reduced moment, somewhat greater savings may be obtained.

(*c*) *Beams Made from Structural Tees.*—A third method of obtaining an unsymmetrical section is by welding together the webs of two structural tees of different sizes, and thus producing an I-shaped beam with one flange heavier than the other. Several beams of this type were designed using structural tees cut from wide-flange sections, and the average saving in weight as compared to composite rolled beams was 11%. This is less than one half as much saving as can be obtained with all-welded beams. Comparisons with other types of beams may be made by reference to Col. 7, Table 3.

5. Rolled Unsymmetrical Beams

The comparatively large saving in weight which may be obtained with the use of unsymmetrical beams in composite construction suggests the desirability of rolling unsymmetrical beams having one flange heavier than the other. A brief study has been made of the optimum dimensions of such beams and of the economies that might be obtained from their use in composite construction.

An unsymmetrical rolled beam permitting the greatest saving in weight would have dimensions and proportions similar to those of the aforementioned built-up all-welded beams. The saving in weight possible from the use of such welded beams in place of ordinary rolled beams was about 26%. The saving in weight for a series of unsymmetrical rolled beams patterned after the welded beams would be somewhat less because of the lack of flexibility in the choice of a beam to provide a given section modulus. There is some question, however, as to whether beams of these proportions could be rolled because of the relatively large difference in thickness of the web and the flanges.

A more practical unsymmetrical rolled beam suitable for use in composite construction may be obtained by a relatively simple modification of the ordinary wide-flange beam, as follows: The area of the top flange is reduced by one half and the remaining dimensions are unchanged. The weight of a beam so modified will be reduced by approximately 14%. However, because the series of beams is not continuous, the average savings in weight arising from the use of such beams in place of ordinary rolled beams will be about 11%, as indicated in Col. 6, Table 3.

6. Composite Bridges with Temporary Supports

Since the section modulus of a composite beam is always appreciably greater than that for a rolled beam alone, further economies in the weight of the beam may be obtained by constructing the bridge in such a manner that the dead load as well as the live load is carried by the composite section. This result can be obtained by placing temporary supports or shores beneath the beams and removing them only after the concrete of the slab has set.

The additional saving in weight resulting from the use of temporary supports averaged about 16% for rolled beams, with or without cover plates, as indicated in Cols. 3 and 5, Table 3. For built-up welded beams the additional saving ranged from 16% for short spans to as much as 40% for long spans where appreciable reductions in the area of the top flange could be made. These data are given in Col. 9, Table 3.

In obtaining the foregoing results, it was assumed that the beams were supported along their entire length although in practice the beams would probably be supported by falsework bents at only two or three points. The error resulting from this assumption was investigated for one, two, or three equally spaced temporary supports. For one support, the maximum dead-load stress in the bottom flange of the beam was 3% to 6% less than that computed for the assumption of continuous support, while for two or three supports, the corresponding stress was 1% to 3% greater.

PART B. PROBLEMS IN THE DESIGN OF COMPOSITE BEAMS FOR I-BEAM BRIDGES

A number of studies, both experimental and analytical, have been made of various problems encountered in the design of composite beams and composite I-beam bridges. Since frequent reference is made to experimental data, it is necessary to give a brief description of certain of the laboratory tests that have been made.

1. Tests of Composite I-Beam Bridges

Laboratory tests have been made on twenty quarter-scale models of simple-span I-beam bridges, nine of which were of composite construction utilizing channel shear connectors. Of these nine, six were right bridges with prototype spans of 20 ft and 60 ft, and three were skew bridges with prototype spans of 60 ft and angles of skew of 30° and 60°.

In these tests,[13] the model bridges were loaded with either two or four concentrated loads simulating the rear wheels of one or two trucks. Measurements made included strains in the beams and in the slab reinforcement, deflections of the beams, and slip between the slab and the beams. The results of the tests for the various bridges were compared with each other and with the analysis for the simple-span right bridge.

2. Tests of Composite T-Beams

Tests were made on a number of small composite T-beams of the type illustrated in Fig. 23. These T-beams consisted of an I-beam and a mortar slab, tied together with channel shear connectors, 1 in. by $\frac{3}{8}$ in. by 1 in. long,

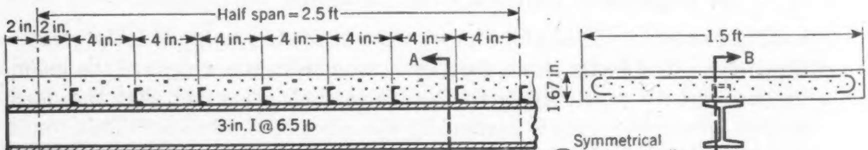


FIG. 23.—DETAILS OF BEAM SPECIMEN, T-BEAM TESTS OF SHEAR CONNECTORS

welded to the top flange of the I-beam and embedded in the slab. The variables studied included the size and spacing of the shear connectors and the compressive strength of the mortar slab. The beams were loaded with a single concentrated load, usually at midspan, and measurement was made of deflection, slip between the slab and I-beam, and strains in both the I-beam and the slab.

3. Slip Between Slab and Beams in I-Beam Bridges

A fundamental question in the design of composite beams is whether or not the transformed area method may safely be used in the calculation of stresses. The principal condition for the use of this method is that there be no slip between the two elements making up the beam. The degree to which this condition was obtained in the tests of composite I-beam bridge models with channel shear connectors is illustrated by the typical curves in Fig. 24.[25] The slip

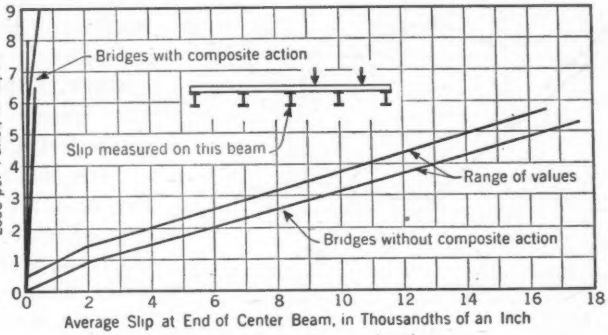


Fig. 24.—Load-Slip Curves for Tests with Simulated Wheel Loads

between the slab and the center beam at the ends of the beam is shown in Fig. 24 for both composite and noncomposite model bridges having span lengths of 5 ft; the results, however, are typical of all the bridges tested. The maximum slip obtained in the bridges with composite construction was less than 0.0005 in., even at loads several times greater than those ordinarily considered in design.

4. Effect of Slip on Strain in Beams

The effect of small amounts of slip on the distribution of strain in a composite beam is shown in Fig. 25, which is based on data obtained in tests of the small composite T-beams which were similar to those used in the 5-ft I-beam bridges. The distribution of strains throughout the depth of the composite beam is shown in Fig. 25. The dashed line represents the theoretical straight-line distribution for an ideal composite beam with no slip between the two elements. The solid circles represent measured strains, and the solid line drawn through them represents the strain distribution existing in these beams when some slip was present. The amount of slip at the location of the measurements is also shown.

The slip for beam T1 was approximately the same as that indicated in Fig. 24 for the model bridges. The agreement with the theoretical distribution is quite good, and the increase in strain in the bottom flange—the most important value—is negligible.

[25] "Studies of Slab and Beam Highway Bridges: Part I. Tests of Simple-Span Right I-Beam Bridges," by N. M. Newmark, C. P. Siess, and R. R. Penman, *Bulletin No. 363*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., 1946, p. 43, Fig. 13.

Beam T2 was similar to beam T1 except that the compressive strength of the mortar slab was considerably less. As a result, there was a greater amount of slip in this beam, and its effect can be noted in the jog in the strain-distribution curve at the junction of the slab and beam. However, the increase in bottom-flange strain is still quite small.

The shear connectors in beam T3 were considerably weaker than those used in beams T1 and T2, and appreciably greater slips were obtained. The effect of slip on the strains, however, is greatest at the junction of the beam and slab, and is still not important at the bottom of the beam where the greatest strains occur.

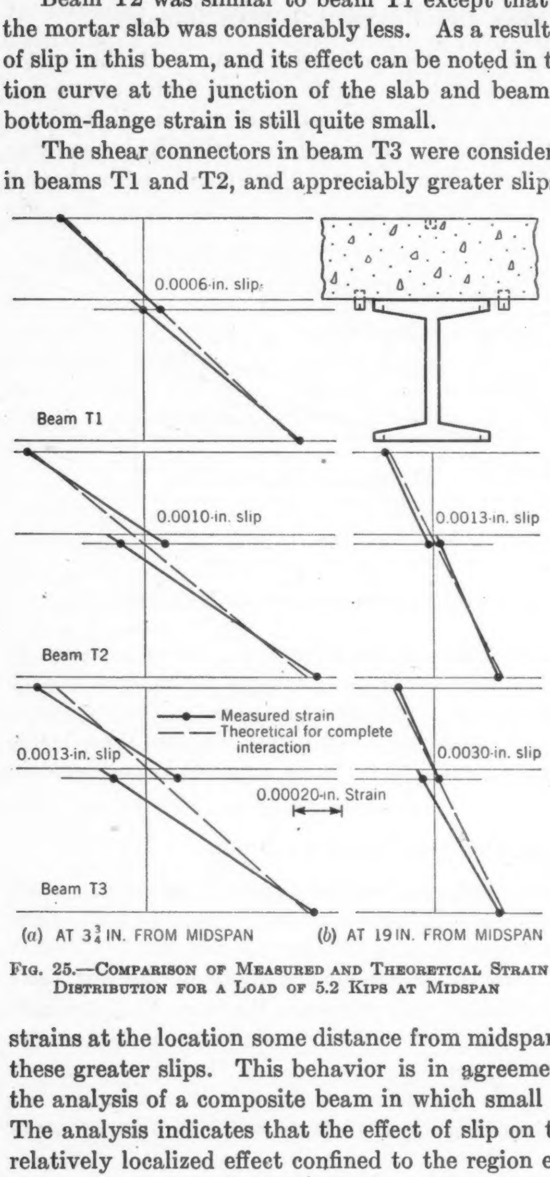


FIG. 25.—COMPARISON OF MEASURED AND THEORETICAL STRAIN DISTRIBUTION FOR A LOAD OF 5.2 KIPS AT MIDSPAN

The data plotted in Fig. 25(*a*) are for a section only $3\frac{3}{4}$ in. from the point of load application at midspan. Those in Fig. 25(*b*) are for a section near the quarter point of the span. In these tests, the slip was greatest at the ends of the beam and decreased in magnitude for points closer to the location of the load at midspan where, of course, it was zero. It may be noted in Fig. 25 that the slips at a section 19 in. from midspan were greater than those at the section near the load point. On the other hand, it may also be noted that the strains at the location some distance from midspan are affected only slightly by these greater slips. This behavior is in agreement with that predicted from the analysis of a composite beam in which small amounts of slip are present. The analysis indicates that the effect of slip on the distribution of strain is a relatively localized effect confined to the region extending a short distance on either side of the point of application of a concentrated load.

The conclusion can be drawn from the foregoing results that a slab and beam tied together by suitable shear connectors may be considered to act as a single composite section, and may therefore be analyzed on the basis of the transformed section.

5. Effective Width of Slab as T-Beam Flange

For composite I-beam bridges of ordinary dimensions, it has been recommended that a section of the slab having a width equal to the beam spacing be assumed to act as part of the composite beam. If this assumption is to be valid, the distribution of compressive stress in the slab acting as a flange of the T-beam should be uniform over the entire width. The distribution of this

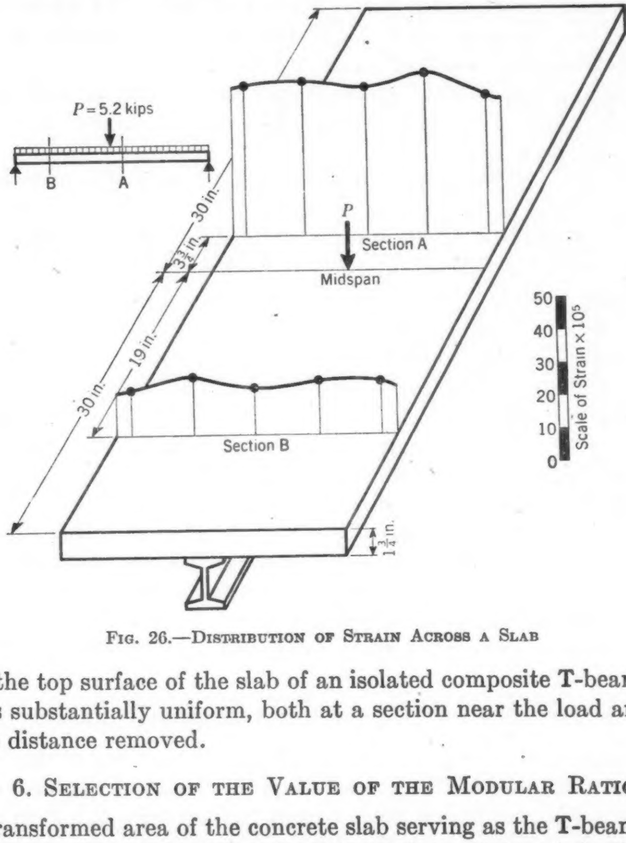


Fig. 26.—Distribution of Strain Across a Slab

stress on the top surface of the slab of an isolated composite T-beam, shown in Fig. 26, is substantially uniform, both at a section near the load and at a section some distance removed.

6. Selection of the Value of the Modular Ratio

The transformed area of the concrete slab serving as the T-beam flange is a direct function of the value of n, the ratio of the modulus of elasticity of steel to that of concrete. It is perhaps fortunate, therefore, that the properties of the transformed section of the composite beam are not appreciably affected by moderately large variations in the value of this ratio. For example, a decrease of 28% in the value of n (from 10 to 7.15) produces a decrease of only 3% in the section modulus of the bottom flange of the composite beam, a decrease of 7% to 9% in the moment of inertia, and a decrease of 11% in the magnitude of the horizontal shear between the slab and the beam for a given value of vertical shear. These changes are negligible and a value of n equal to 10 may safely be used in computing the properties of composite beams, except in cases where it is desirable to take into account the effects of plastic flow of the slab.

7. Plastic Flow of Slab

If a composite bridge is constructed without the use of shoring beneath the beams, the dead load is carried by the steel beams alone and the concrete slab acts only for live loads. Consequently, there should be no significant plastic flow in the slab. On the other hand, if shoring is used and the composite beam carries both dead and live load, the slab is subjected to a sustained compressive stress, and plastic flow is likely to occur. The magnitude of this flow will not be very great since the maximum compressive stress in the slab will seldom exceed 600 lb per sq in. or 700 lb per sq in. However, if it is desired to make allowance for the effects of such flow, it may be done by computing the contribution of the slab on the basis of an increased value of n. A commonly used value is $n = 30$.

The effect of considering $n = 30$ rather than $n = 10$ is to decrease the section modulus of the bottom flange by about 10%. A much greater decrease in the top flange section modulus will be produced, but in general this will be of little importance since the stresses in the beam at that location are quite small if shoring is used. The decrease in moment of inertia of the composite section is about 25%, and for this reason it may be desirable to consider plastic flow when computing dead-load deflections.

A decrease of about 25% in the magnitude of the horizontal shear between the beam and the slab will also be obtained as the result of using $n = 30$ instead of $n = 10$. However, since full plastic flow will not be realized for some time, and possibly not at all, the shear connectors should be designed for the greater horizontal shear computed on the basis of $n = 10$.

A further possibility for plastic flow occurs in a bridge constructed without shoring if a wearing surface is added at some time after construction is completed. The weight of this added material is carried by the composite section and thus produces sustained stresses in the slab. The effect of possible plastic flow in such a case, however, is quite small and may usually be neglected in design.

8. Methods of Proportioning Composite Beams

In most cases in the design of I-beam bridges, the spacing of the beams and the thickness of the slab will be known when the design of the beam is undertaken. The problem of design then reduces to the choice of a steel beam section which, in combination with the slab, will be able to carry the known moments at the specified allowable stresses. The procedure for doing this, however, is usually neither simple nor direct, especially in cases where temporary supports are not used during construction. Although, in general, a cut-and-try procedure is required, in certain cases that have been studied in connection with this investigation, design aids have been developed which materially reduce the labor required. Two types of such aids are: (*a*) Those facilitating the design of composite rolled beams without cover plates; and (*b*) those for use in the design of built-up welded beams.

(*a*) *Designs with Rolled Beams.*—For composite beams utilizing rolled wide-flange beam sections without cover plates, the problem of choosing a section

may be simplified by the use of tables giving the properties of composite beams Such tables have been prepared for composite beams consisting of various wide-flange beam sections acting in combination with a section of slab varying in width from 5 ft to 7 ft, and in thickness from 6 in. to 8 in. The properties tabulated are: (1) Section modulus of the bottom flange; (2) moment of inertia of the beam; and (3) ratio of horizontal shear at the junction of slab and beam to the vertical shear at any given location.

(b) *Designs with Built-Up Welded Beams.*—Part A of this paper demonstrated that the built-up welded beam with unsymmetrical flanges is a particularly economical type for use in composite construction. The most economical proportions for such a beam will be those for which the total dead-load plus live-load stresses are equal for the two flanges. If temporary supports are not used, the adjustment of flange areas to give equal stresses is rather complicated, since dead load is carried by the steel beam alone and live load is carried by the composite section. However, for a particular set of conditions, the process of design may be simplified by the use of an empirical procedure based on the results of a number of designs made by trial and error. In these designs, the webs were made of such a thickness that stiffeners were not required. Computations indicated that a thickness of not less than 1/87 times the clear depth would provide adequate safety against buckling. Any appreciable change in this ratio will require alteration of the following empirical formulas for the trial flange areas.

The procedure developed consists of a preliminary design, an analysis, and a revision. Usually a second analysis will not be necessary unless unusually high precision is required.

(1) Preliminary Design.—The area of the bottom flange, A_b, and the area of the top flange, A_t, in square inches, are given by the expressions:

$$A_b = A_o - 2.5 \qquad \text{(11a)}$$

and

$$A_t = 0.8\,A_o - 6.0 \qquad \text{(11b)}$$

in which

$$A_o = \frac{M_D + M_L}{f_s \times d_e} \qquad \text{(12)}$$

and $M_D + M_L$ is the sum of dead-load and live-load moments, in inch-pounds; f_s is the allowable stress in steel, in pounds per square inch; and d_e is the distance between centers of gravity of the flanges, in inches, plus the height of any fillet between the slab and the beam.

(2) Analysis.—The stresses are computed for the section obtained in the preliminary design, step (1). It is assumed that the dead-load moment is carried by the steel beam alone and that the live-load moment is carried by the composite section.

(3) Revision.—If the stress in either or both flanges is different from f_s, the flange area may be adjusted in the following manner: Let f_b be the desired stress in the flange; f'_b be the actual stress in the flange; A_b be the desired area of flange to give a stress of f_b; A'_b be the present area of the flange giving a

stress of f'_b; and A_w be the total area of the web. Then

$$\frac{A_b + 0.4\,A_w}{A'_b + 0.4\,A_w} = \frac{f'_b}{f_b} \quad \text{(13)}$$

This adjustment can be made simultaneously and independently for both top and bottom flanges.

As an illustration of the effect of changing the thickness-depth ratio for the web, the following expressions for trial flange areas have been found to hold approximately if this ratio is 1/50 instead of 1/87 as before:

$$A_b = 0.95\,A_o - 2.5 \quad \text{(14a)}$$

and

$$A_t = 0.80\,A_o - 7.5 \quad \text{(14b)}$$

PART C. BEHAVIOR OF SHEAR CONNECTORS FOR COMPOSITE I-BEAM BRIDGES

1. Requirements for Shear Connectors

Composite action is produced in an I-beam bridge by the introduction of shear connectors whose function is to tie together the concrete slab and the steel I-beam in such a manner that they act as a single element. To do this, two requirements must be met by the shear connectors: First, they must effectively prevent slip between the slab and the beam; and, second, they must be strong enough to withstand the shearing forces with an adequate factor of safety.

The capacity of shear connectors to prevent slip has been demonstrated by the results of tests on quarter-scale model bridges discussed in Part B.

When slip is prevented, the shear connectors are subjected to loads resulting from the shearing forces acting between the slab and the beam. The total force acting on each connector may easily be computed from the horizontal shear and the spacing of the connectors. However, the stresses in the connector cannot be obtained unless the distribution of force over the height and width of the connector is known. A primary objective of this investigation was the determination of the nature of this distribution and of the factors that may cause changes in it.

Another factor which must be considered is that the shear connectors in many bridges are subjected to repeated loading under conditions favorable to failure in fatigue. In a composite bridge constructed without shoring beneath the beams, the shear connectors are stressed as a result of live loads only, since dead load is carried by the steel stringers acting alone. At the center of the span where a complete reversal of shear occurs for moving loads, the stress on the connectors also undergoes complete reversal. At the ends of the span, the stress varies from zero to a maximum with each passage of a load. This large range of stress, plus the possible presence of stress raisers at or near welds or fillets, makes it desirable to consider the possibility of failure under repeated loading.

Consideration must also be given to the stresses produced in the concrete slab in bearing against the shear connectors. It is apparent that the function

of the connector may be impaired or destroyed by the failure of the concrete adjacent to it, as well as by the failure of the connector itself.

A third requirement of shear connectors which is desirable, although not necessarily essential, is that they be able to resist uplift forces tending to pull the slab vertically away from the beam. This tendency for separation may be the result of loading, or it may be caused by warping of the slab resulting from shrinkage, unequal expansion due to radiant heat, or other effects.

2. Bond As Shear Connection

Both laboratory tests and field observations have shown that bond between the steel beam and the concrete slab may provide a fairly effective shear connection under certain conditions. However, once this bond is broken it cannot be restored, and its function as a shear connection is lost forever. Although the bond stresses resulting from loads may be fairly low, there is always the possibility that the bond may be destroyed as a result of movements caused by shrinkage, temperature, vibration, etc. In particular, bond offers practically no resistance to vertical separation of the slab and beam, and such separation destroys entirely its ability to transfer shear.

There can be no question that bond alone should not be relied on as a shear connection for beams which are designed for composite action. A positive mechanical method for transferring shear and preventing slip and uplift should be provided. The use of a positive shear connection also helps to preserve the bond intact by holding the slab and beam in contact.

3. Push-Out Tests of Shear Connectors

To compare the behavior of several different types of shear connectors, a series of simple shear tests was made, utilizing the push-out type of specimen illustrated in Fig. 27. One shear connector was welded to each flange of a

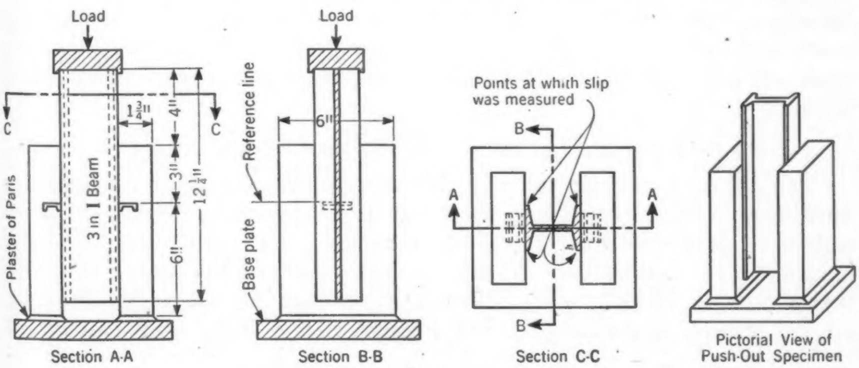


Fig. 27.—Details of Push-Out Specimen

short length of 3-in. I-beam, and each of the connectors was embedded in a mortar slab cast against the flange of the beam. The connectors were subjected to shear by applying load to the beam and forcing it to move relative to the slabs. Slip between the I-beam and the slabs was measured with dial

indicators at the four corners of the I-beam as indicated by the arrows on section C–C, Fig. 27.

These tests included only the types of shear connectors that transmit shearing force primarily by bearing of the connector on the concrete. For some of these types, the width and the thickness of the connector were varied, and in another series of tests, the compressive strength of the mortar in the slabs was varied.

4. Flexible and Rigid Connectors

The various types of connectors used in the push-out tests may be divided into two general classes, rigid connectors and flexible connectors. The types considered as rigid consisted of short lengths of channel, angle, or tee section placed on end with their longitudinal axis perpendicular to the beam flange. Several connectors of the flexible type were tested; the only one considered herein, however, is the channel placed with the web vertical and having one flange welded to the beam.

Typical load-slip curves for both rigid and flexible connectors are shown in Fig. 28(*a*). It is evident from these curves that the rigid type of connector is superior to the flexible-type channel, from the standpoints of both ultimate strength and the amount of slip permitted at low loads. It is significant, however, that the differences in behavior of the two types are much less than might be expected from the very large differences in strength and stiffness of the connectors themselves. The reason for this is that both the ultimate strength of the test specimens and their load-slip characteristics were dependent to a very considerable extent on the stresses and deformations in the concrete, and were affected to a much smaller degree by the properties of the shear connectors.

Although the rigid type of connector has certain advantages, as illustrated by the results of these tests, this type also possesses certain disadvantages, the most important of which is its failure to resist uplift forces tending to separate the slab and the beam. Another disadvantage results from the relatively large width of the connectors in the direction parallel to the beam axis, which in many cases will offer serious interference to placing the transverse reinforcement in the slab.

5. Effect of Variations in Thickness of Connector

The effect of varying the web thickness of channel shear connectors is illustrated in Fig. 28(*b*). Load-slip curves are given both for ordinary channels and for channels with the top flange removed. Two web thicknesses were used: 0.125 in. and 0.097 in. It may be concluded from the data in Fig. 28(*b*) that a decrease of 22% in the thickness of the web did not produce any significant changes in the behavior of the connectors.

6. Effect of Variations in Compressive Strength of Slab

The load-slip curves in Fig. 28(*c*) are for specimens with three different values of compressive strength. It is evident from the curves that the effectiveness of the shear connectors was increased considerably by an increase in the compressive strength of the slabs. This increase, however, was proportionately less for the higher strengths than for the lower ones.

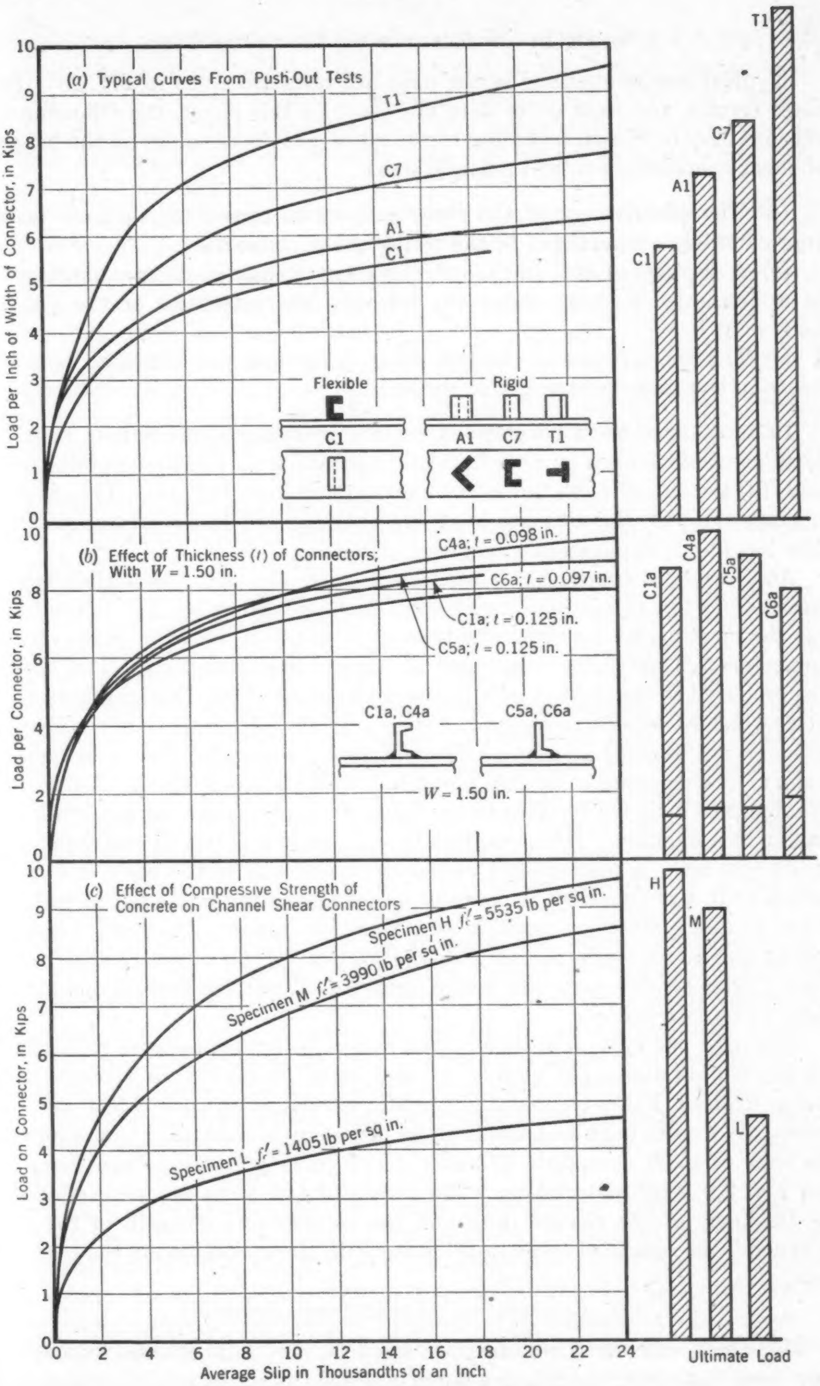


FIG. 28.—COMPARISON OF LOAD-SLIP CURVES

7. Summary of Results of Push-Out Tests

Typical results obtained in the push-out tests are given in Fig. 28. From these results, and from other data not given in this paper, the following conclusions may be drawn regarding the effect of certain variables on the behavior of shear connectors loaded in simple shear:

(*a*) The effectiveness of the shear connectors considered in these tests is approximately proportional to the width of the connector;

(*b*) A decrease of 22% in the web thickness of channel shear connectors had no appreciable effect on either the load-slip characteristics or the ultimate load; and

(*c*) The effectiveness of channel shear connectors was increased by an increase in compressive strength of the slab.

In addition to these conclusions, certain characteristics common to all the shear connectors may be noted: (1) Slip begins with the first application of load; (2) the load-slip relation is not linear at any load, but is roughly parabolic in shape; and (3) the ultimate loads are accompanied by such large slips that they have little application as a design criterion.

As a result of the push-out tests, it was decided that the flexible type of shear connector, consisting of a channel with one flange welded to the beam, was the most promising type, and that all future tests would be made on such connectors. This choice was based on several considerations. The channel was preferred to the rigid type of connector because of the disadvantages of the latter which have been mentioned. Moreover, the performance of the channel, although not as good as that of the rigid types, was still believed to be satisfactory. Other types of flexible connectors were also considered. A **Z**-section, although not included in these tests, would probably behave in a manner very similar to a channel. However, this type of section is not so easily procured as the channel. A plate placed vertically and welded to the beam is a lighter section than a channel but it provides no resistance to uplift and, in addition, is not so easily placed in position and welded in place. The latter objection applies also to an angle placed with the vertical leg down and welded to the beam. This type of connector resists uplift forces, but the welding operation is likely to be awkward.

The push-out type of specimen was chosen primarily because it furnished a fairly simple and rapid method of comparing several types of connectors. It was realized, however, that the conditions existing in such a test were not necessarily similar in all respects to those in a composite beam. Consequently, the tests of small composite **T**-beams which have previously been described (see Fig. 23) were undertaken. The effects of two variables were studied—the thickness of the channel web and the compressive strength of the slab. In general, the results of these tests agreed with those obtained in the push-out tests.

8. Concept of Dowel-Like Action

The most interesting results from the tests on channel shear connectors were those indicating the relative importance of variations in web thickness of

the channel and variations in compressive strength of the slab. A study of these results led to the conclusion that the action of this type of connector is similar to that of a dowel embedded in an elastic medium. The type of behavior associated with this concept of dowel-like action is illustrated in Fig. 29. A possible distribution of pressure on the dowel or connector is shown in Fig. 29(*a*). It is markedly nonuniform and may change sign as indicated.

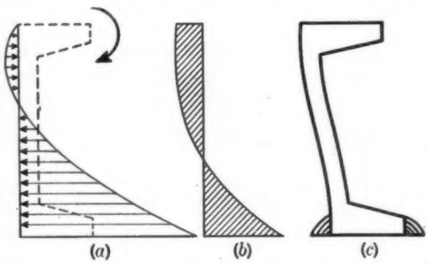


Fig. 29

The exact shape of the pressure distribution depends on the relative stiffnesses of the connector and the concrete. The stiffness of the channel depends primarily on its web thickness; the stiffness of the concrete depends on its modulus of elasticity and thus on its compressive strength. Variation of either the web thickness of the channel or the strength of the concrete changes the relative stiffness of the dowel and matrix and thereby changes the pressure distribution.

Both the maximum stress in the concrete and the moment in the connector at the critical section are functions of the shape of the pressure distribution curve. For example, if the thickness of the channel web is reduced by one half, the section modulus of the channel is reduced to one fourth of its original value. However, its stiffness is only one eighth of its original value, and the relative stiffness of the connector as compared to that of the concrete is greatly decreased. This decrease, in turn, is accompanied by an increase in nonuniformity of the pressure distribution and a lowering of the center of pressure which decreases the moment on the channel at the critical section. Thus, a reduction in web thickness decreases the moment as well as the section modulus, and the corresponding decrease in effectiveness of the connector may be relatively small.

The effect of variations in concrete strength may also be considered. If the concrete strength is increased, there is a corresponding increase in modulus of elasticity which has the effect of decreasing the stiffness of the connector as compared to that of the concrete. As before, this change results in a more nonuniform pressure distribution, and a consequent increase in the maximum concrete bearing stress. Thus an increase in concrete strength is accompanied by an increase in concrete stress, and the net result is an increase in effectiveness of the connector somewhat less proportionately than the increase in strength.

A curve of moment in the connector corresponding to the assumed pressure distribution is shown in Fig. 29(*b*). The moment indicated at the upper end of the connector is a result of the restraint to rotation offered by the flange of the channel. To apply the dowel analogy, the moment is that which would exist if the flange were bent 90° and extended as a continuation of the web.

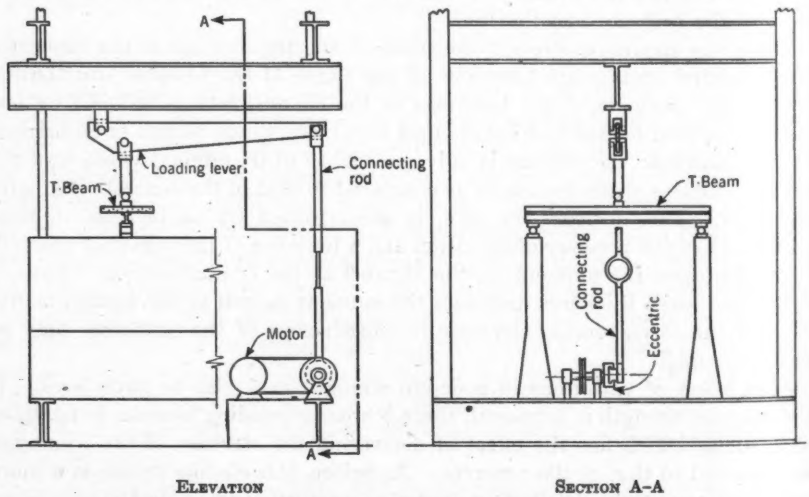
Fig. 29(*c*) represents a type of deflection consistent with the pressure and moment curves shown. Deflections of a typical channel shear connector,

after being subjected to shear in a push-out test, are in substantial agreement with the approximate deflection shown in Fig. 29(*c*).

The hypothesis of dowel-like action of channel shear connectors serves to explain qualitatively the observed behavior of such connectors in both the push-out tests and the static tests of composite beams. It also suggests that rather high compressive stresses are produced in the concrete, and probably also high bending stresses in the channel web. These potentially high stresses, together with the fact that the shear connectors in an I-beam bridge are subjected to repeated loads, indicated the desirability of fatigue tests on channel shear connectors. Moreover, it was felt that such tests might provide a quantitative basis for evaluating the effects of variation in concrete strength and web thickness.

9. Repeated-Load Tests of Channel Shear Connectors

It was decided that fatigue tests of shear connectors could be made most conveniently by applying repeated loads to small composite T-beams of the type illustrated in Fig. 23. Such tests have been made in the testing machine illustrated in Fig. 30. Load is applied at midspan of the T-beam by a lever



Elevation

Section A–A

Fig. 30.—Machine for Applying Repeated Load to a Composite Beam

actuated by a connecting rod attached to a variable-throw eccentric. The load is measured by an elastic-ring dynamometer inserted in the connecting rod. The speed of operation is 190 cycles per min. Measurements made during the repeated-load test include slip between the slab and the I-beam at the ends of the beam, and deflection of the beam at midspan. Static tests in which strains in the beam are also measured are made on each beam both before and after it is tested in fatigue.

The principal variables considered in these tests were the compressive strength and modulus of elasticity of the mortar slab and the web thickness of the channel shear connectors.

There is little information available regarding the number of load applications that should be considered for highway bridges. However, a preliminary study of traffic survey and "loadometer" data indicated that the number of heavy truck loads may easily approach 1,000,000 or 2,000,000 during the life of a bridge. Consequently, these tests were planned to obtain data for load-cycle diagrams with a maximum of 2,000,000 cycles.

10. Manner of Failure in Fatigue

A rather detailed description of the manner of failure of the composite T-beams in the repeated-load tests is desirable so that the results of these tests might be interpreted properly. There was no well-defined point at which it could be stated that failure had occurred. Instead there was a gradual, progressive breakdown of composite action accompanied by increases in slip and deflection. Finally, the slip became so great that it was obvious that interaction between the beam and the slab was seriously impaired. In these tests, this stage of failure was usually reached when the slip at the end of the beam attained a value of 0.010 in. If the test was stopped at this point, it was found in every case that several shear connectors at one end of the beam had fractured.

In these tests, all the shear connectors were faced in the same direction, as indicated in Fig. 31(a). As a result, the direction of the load in relation to

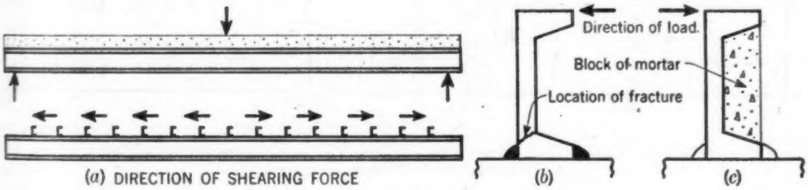


Fig. 31.—Failure of Channels in Repeated-Load Tests

the direction in which the channel flanges projected was different for the two ends of the beam. With few exceptions, the fractured connectors were those loaded in the manner shown in Fig. 31(b). It is probable that the conditions at the other end of the beam were as illustrated in Fig. 31(c)—that is, the channel was strengthened by the resistance to compression of the block of mortar enclosed between its flanges.

In all cases, the final failure of the beam was by fracture of the shear connectors at the location shown in Fig. 31(b). There is some question, however, as to whether this fracture was always the primary cause of failure. If the concrete adjacent to the connector fails by crushing, there will be a redistribution of pressure on the connector in such a manner as to raise the center of pressure and consequently to increase the moment on the channel. This increased moment, in turn, may produce a fatigue failure of the connector. There is no way of knowing conclusively, from the data thus far obtained, whether the primary cause of failure in any given case is crushing of the concrete or fracture of the connector.

11. Effect of Variations in Concrete Strength

A series of repeated-load tests was made on twenty composite T-beams for which the compressive strength of the mortar slab ranged from 1,000 lb per sq in. to 6,500 lb per sq in. The results of those tests are given in Fig. 32. In Fig. 32(*a*) the load on the T-beam (not corrected for compressive strength) is plotted against the logarithm of the number of cycles required to produce failure. The beams are divided into three groups on the basis of their compressive strengths, as indicated by different symbols. The data contained in

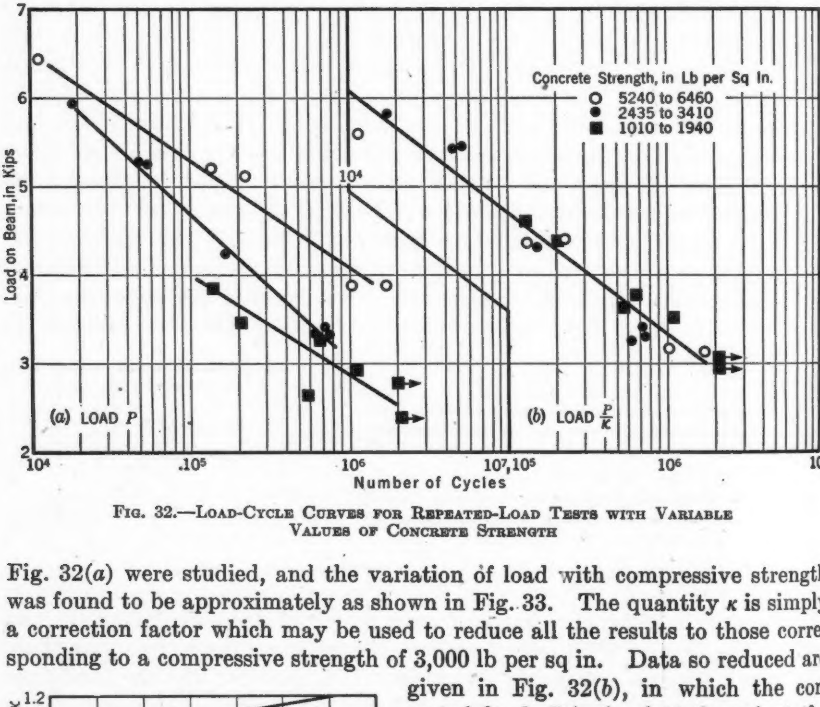


FIG. 32.—LOAD-CYCLE CURVES FOR REPEATED-LOAD TESTS WITH VARIABLE VALUES OF CONCRETE STRENGTH

Fig. 32(*a*) were studied, and the variation of load with compressive strength was found to be approximately as shown in Fig. 33. The quantity κ is simply a correction factor which may be used to reduce all the results to those corresponding to a compressive strength of 3,000 lb per sq in. Data so reduced are given in Fig. 32(*b*), in which the corrected load, P/κ, is plotted against the number of cycles.

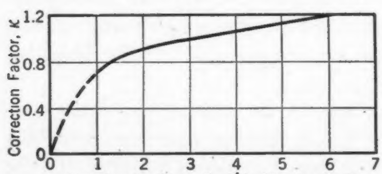


FIG. 33.—BASIS OF CORRECTION FOR COMPRESSIVE STRENGTH

The effect of compressive strength on the effectiveness of channel shear connectors is defined quantitatively by the curve in Fig. 33. Precisely why this relation takes the form shown cannot be determined conclusively from the available evidence. However, a possible explanation may be that at low compressive strengths the primary cause of failure is crushing of the concrete, whereas at higher compressive strengths the primary cause is fracture of the steel without prior failure of the concrete. Since the strength of the concrete affects its stiffness and thus affects the shape of the stress distribution, there

would still be some variation in load with concrete strength for the higher strengths even though primary failure was by fracture.

12. Effect of Variations in Web Thickness

Repeated-load tests were also made on thirty-three beams having connectors with different web thicknesses. Two values of web thickness were used—$\frac{1}{8}$ in. and $\frac{1}{16}$ in.— and for each thickness two concrete strengths—approximately 2,000 lb per sq in. and 4,000 lb per sq in.—were used. The results are presented in Fig. 34 in the form of load-cycle curves for the connectors of each web thick-

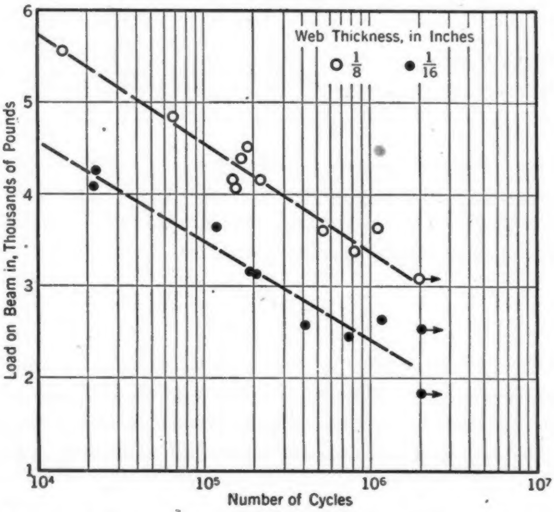


Fig. 34.—Effect of Web Thickness on Effectiveness of Channel Shear Connectors

ness. The variations in compressive strength have been eliminated by a relation similar to that given in Fig. 33.

It may be determined from the diagram that a reduction in web thickness from $\frac{1}{8}$ in. to $\frac{1}{16}$ in. reduced the effectiveness of the connectors by only about 25%. Since the reduction in section modulus of the channel web is 75%, the decrease in thickness must have been accompanied by an appreciable reduction in moment on the connector. In accordance with the dowel concept, such a reduction would result from the change in pressure distribution caused by the reduction in stiffness of the connector.

13. Distribution of Pressure on Channel

It has been shown that the assumption of dowel-like behavior of channel shear connectors is in reasonably good agreement with the results of tests, and this hypothesis serves to explain the influence of certain variables on the effectiveness of the connectors. The principal assumption of this hypothesis is that the distribution of pressure on the connector is a function of the relative stiffness of the connector and the concrete. The distribution of this pressure

is quite complex, and it is doubtful if its true nature can be determined by tests similar to those which have been described. Nevertheless, it is possible to draw a greatly simplified and idealized pressure diagram which has certain properties similar to those of the actual distribution. Such a pressure diagram is given in Fig. 35. Its chief characteristics are the uniform distribution of pressure over the very stiff area of the channel opposite the flange, and the expression of the distance h_w as a function of the web thickness t.

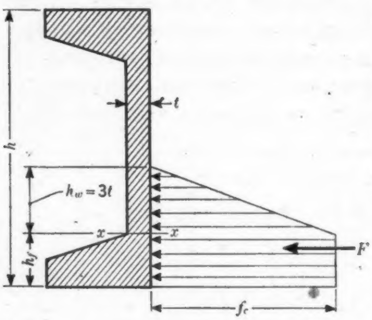


FIG. 35.—IDEALIZED PRESSURE DISTRIBUTION

Let F be the total load, in pounds per inch of width. Then, for the stress distribution of Fig. 35, the maximum concrete stress is expressed by

$$f_c = \frac{F}{h_f + \frac{3\,t}{2}} \quad \text{.......(15a)}$$

and the maximum steel stress on section x–x by

$$f_s = \frac{9\,F}{h_f + \frac{3\,t}{2}} \quad \text{.......................(15b)}$$

This type of pressure distribution is being studied as a possible basis for the design of channel shear connectors. However, sufficient information is not yet available to permit recommendations regarding a definite design procedure to be made.

SUMMARY

This paper has presented the results of analytical studies and laboratory tests of composite construction for I-beam highway bridges. The principal conclusions can be summarized as follows:

a. Savings in Weight with Composite Construction.—Composite construction permits the use of lighter beams and at the same time provides a much stiffer structure. The saving in weight which may be obtained by the use of various types of construction ranges from 8% for ordinary rolled beams to 30% or more for built-up welded beams.

b. Design of Composite. Beams.—A composite beam consisting of a slab and beam tied together by a suitable shear connector may be analyzed and designed on the basis of the transformed section. In I-beam bridges of ordinary proportions, a section of the slab having a width equal to the beam spacing may be considered effective as part of the composite beam.

Variations in the value of n, the ratio of modulus of elasticity of steel to that of concrete, have only a negligible effect on the properties of a composite beam. Except where it is desired to allow for effects of plastic flow, it is sufficiently accurate to assume a value of $n = 10$.

c. Shear Connectors.—This investigation was concerned chiefly with the behavior of channel shear connectors. The effects of the following variables were investigated—width of connectors, thickness of channel web, and compressive strength of the concrete in the slab. The principal results were as follows:

(1) The effectiveness of channel shear connectors was approximately proportional to the width of the connector;

(2) Variations in web thickness produced corresponding variations in the effectiveness of the connectors, but the magnitudes of these changes were far less than would be expected from the changes in section modulus of the channel web; and

(3) The effectiveness of channel shear connectors was increased by an increase in compressive strength of the concrete, but by a proportionately smaller amount for the higher compressive strengths.

The effects of changes in web thickness and concrete strength on the behavior of channel shear connectors may be explained by the concept of dowel-like action. According to this concept, the nature of the distribution of bearing pressure on the connector is a function of the relative stiffness of the channel and the concrete.

Acknowledgments

The comparative design studies in Part A were made by C. A. Erzen and N. Y. Yakovljevitch, graduate students in civil engineering, and M. N. Tokay, special research assistant in theoretical and applied mechanics. The repeated-load tests of shear connectors were made in part by R. F. Mosher, Jun. ASCE, and R. A. Bennitt, Jr., graduate students in civil engineering, and W. E. Johnson, special research associate in civil engineering.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

RIVER INFILTRATION AS A SOURCE OF GROUND WATER SUPPLY

Discussion

BY C. V. YOUNGQUIST

C. V. YOUNGQUIST,[3] M. ASCE.—A timely and thorough analysis is made for the profession of a water-supply installation, utilizing as its source ground water, which is replenished principally by river infiltration. The writer would like to add some general statements on river infiltration with particular reference to experience in Ohio.

The use of ground water has more than doubled in the decade from 1937 to 1947 because of its increased demand by industry, municipalities, and agriculture. Ground water is desired because of its low temperature, constant chemical quality, clarity, and freedom from pollution. The increased demand has required a more thorough inventory of the ground water resources, particularly those that determine quantities available for further development. Under conditions of ground water occurrence in Ohio the source and rate of ground water recharge is all important if a permanent supply is to be developed.

Investigations of Ohio's ground water resources by the Ohio Water Resources Board in cooperation with the United States Geological Survey have established the fact that the greatest ground water pumpage in Ohio occurs in present stream valleys underlain by permeable glacial gravels. In these areas the principal source of ground water recharge is generally river infiltration, which occurs continuously through the stream bed and, in some instances, through permeable areas in the flood plains when the river is at flood stage.

River infiltration can be ascertained by the following methods: (1) Actual measurement of stream loss through infiltration by current meter in the vicinity of a producing well field, which is possible only when the ground water pumpage is large enough to be an appreciable factor of the stream flow (which, of course was not the case in the example described by the author); (2) correlation of river fluctuations with those of the water table in the pumped area; (3) observation of changes in the temperature of ground water due to infiltration; (4)

NOTE.—This paper by Raphael G. Kazmann was published in June, 1947, *Proceedings*.

[3] Chf. Engr., Ohio Water Resources Board, Dept. of Public Works, Columbus, Ohio.

observation of changes in the chemical quality of water due to infiltration; and (5) a pumping test conducted with properly arranged observation wells indicating a distortion in the shape of the cone of influence where river infiltration occurs. A plotting of drawdown against distance from the pumped well indicates steepening hydraulic gradients in a direction toward the stream from the pumping well as infiltration occurs.

All these methods have been used in Ohio ground water investigations. Although the investigations are not intensive or extensive enough, some quantitative data are available. For example, some direct measurements of river infiltration by current meter at Canton, Ohio, on the middle branch of Nimishillen Creek in the vicinity of the City of Canton's northeast well field, indicate infiltration rates of 3,600,000 gal a day per acre of stream bottom in certain reaches of channel at moderate stream flows. Additional data indicate presumptive evidence of rates of 5,000,000 gal a day per acre or 6,000,000 gal a day per acre at flood stages when water levels in the aquifer have been lowered sufficiently to receive the recharge.

These data are of increasing interest to engineers because they aid in the quantitative analysis of a projected ground water supply with more assurance than is generally believed. The stream-flow record, duration curve, flood-frequency analysis, and mass diagram are used in a manner similar to their use in the design of a surface reservoir. The dimensions and contour of the underground reservoir are ascertained as for a surface reservoir. Additional factors, particularly the physical properties of an aquifer (such as transmissibility and specific yield), must be considered in designing for the utilization of a ground water reservoir which is recharged by infiltration, but in general they are determinable to a fair degree of accuracy.

The underground reservoir permits withdrawals for periods of time at rates greater than the infiltration rate, just as a surface reservoir permits withdrawals greater than the run of the river. This is possible in an infiltration supply with ground water storage because infiltration rates increase with increased river stage. Often a ground water reservoir underlying a stream receives its major recharge during flood stages, in much the same way as a surface reservoir.

One advantage of a ground water reservoir replenished by river infiltration is that useful and valuable land is not removed from productivity by inundation. This is especially important in Ohio where most of the land is valuable and is needed for a growing population.

It is hoped that engineers will awaken to the tremednous possibilities for the utilization of ground water, especially infiltrated supplies. Then, engineers will not allow ground water to remain, as Malcolm Pirnie, M. ASCE, has recently stated, "a neglected natural resource."

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DISCUSSIONS

CONTINUOUS FRAME ANALYSIS BY ELASTIC SUPPORT ACTION

Discussion

BY EDWIN H. GAYLORD, WILLIAM A. CONWELL, AND HARRY GARFINKEL

EDWIN H. GAYLORD,[57] ASSOC. M. ASCE.—An ingenious method for the analysis of continuous frames has been developed by the authors. However, the procedure is handicapped by the somewhat awkward calculations involved, and the writer suggests modifications of the author's equations which he believes lead to fairly simple computations. Flexible connections are excluded in this discussion.

Forming the reciprocal of Eq. 8, after dividing each term by L:

$$\frac{L}{L_{AB}} = \frac{1}{1 + 3\,E\,I\,Z_B/L} = \frac{J_B}{J_B + K} \qquad (90)$$

in which $J_B = 1/Z_B$, and $K = I/L$. (Since continuous frame moments depend on relative moments of inertia, the ratio I/L may be used instead of $3\,I/L$. If different materials are involved, E must be included in the definition of K.) Using the symbol F for the carry-over factor, Eqs. 7 and 90 give

$$F_{AB} = \frac{1}{2}\,\frac{J_{BC}}{J_{BC} + K_{AB}} \qquad (91)$$

The double subscript is used with J to denote the direction from which it is computed. If the division indicated in the parentheses of Eq. 11 is performed, the following equation for J is obtained:

$$J_{AB} = \frac{K_{AB}}{1 - F_{AB}/2} \qquad (92)$$

NOTE.—This paper by J. Charles Rathbun and C. W. Cunningham was published in April, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by L. J. Mensch, Frederick S. Merritt, I. Oesterblom, Thomas C. Kavanagh, A. Floris, and Tao King; November, 1947, by Leroy A. Beaufoy, A. A. Eremin, Robert B. B. Moorman, Eduardo Agramonte, Stephen J. Fraenkel and Robert L. Janes, and Phil M. Ferguson; and January, 1948, by Ralph W. Stewart, and Maurice Barron.

[57] Prof. and Chairman, Dept. of Civ. Eng., Ohio Univ., Athens, Ohio.

Once the values of F and J are computed for a given frame, end moments due to transverse loads can be determined rather simply. Consider the moment at point C in span CE, Fig. 8. If the values of $\bar{x}_A$ and $\bar{x}_B$ as given by Eqs. 17 are substituted in Eqs. 16*a*, the following equation is obtained:

$$M_{CE} = \frac{M_{FCE}\,(4\,L_{CE}\,L - L^2) - 2\,M_{FEC}\,(L_{CE}\,L - L^2)}{4\,L_{CE}\,L_{EC} - L^2} \ldots\ldots (93a)$$

In Eq. 93*a*, the subscript F denotes fixed-end moments. The second term in the numerator is negative to conform to the convention which denotes moments as positive when they are clockwise on the joint (or member). The second term is positive for those who prefer the moment sign convention used in strength of materials. Eq. 93*a* can be written:

$$M_{CE} = 4\,M_{FCE}\,\frac{L - \frac{1}{4}\frac{L^2}{L_{CE}}}{4\,L_{EC} - \frac{L^2}{L_{CE}}} - 2\,M_{FEC}\,\frac{\frac{L}{L_{EC}}\,(L_{CE} - L)}{4\,L_{CE} - \frac{L^2}{L_{EC}}} \ldots\ldots (93b)$$

From Eq. 11,

$$L - \frac{1}{4}\frac{L^2}{L_{CE}} = \frac{3\,E\,I}{J_{CE}} \ldots\ldots (93c)$$

From Eq. 93*c*,

$$\frac{L^2}{L_{CE}} = 4\,L - \frac{12\,E\,I}{J_{CE}} \ldots\ldots (93d)$$

and, therefore,

$$4\,L_{EC} - \frac{L^2}{L_{CE}} = 4\,(L_{EC} - L) + \frac{12\,E\,I}{J_{CE}} \ldots\ldots (93e)$$

Now, from Eqs. 8 and 13*b*,

$$L_{EC} - L = \frac{3\,E\,I}{J_{CA} + J_{CD}} \ldots\ldots (93f)$$

Substituting from Eq. 93*f* into Eq. 93*e*

$$4\,L_{EC} - \frac{L^2}{L_{CE}} = 12\,E\,I\left(\frac{1}{J_{CA} + J_{CD}} + \frac{1}{J_{CE}}\right) \ldots\ldots (93g)$$

The right member of Eq. 93*b* can now be expressed in terms of stiffness factors, by substitution from Eqs. 93*c* and 93*g* into the first term, and, with proper change in notation, by substitution from Eqs. 93*f* and 93*g* into the second term. Finally, observing that $L/L_{EC} = 2\,F_{EC}$, Eq. 93*b* reduces to

$$M_{CE} = M_{FCE}\,\frac{J_{CA} + J_{CD}}{J_{CA} + J_{CD} + J_{CE}} - M_{FEC}\,\frac{F_{EC}\,J_{EC}}{J_{EC} + J_{EF} + J_{EG}} \ldots (93h)$$

Denote by the symbol R the ratio of the stiffness factor of any member to the sum of the stiffnesses of all members framing into the joint. Then, for M_{AB}, Eq. 93*h* may be written:

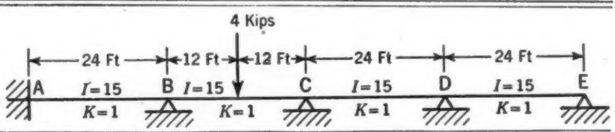
$$M_{AB} = (1 - R_{AB})\,M_{FAB} - F_{BA}\,R_{BA}\,M_{FBA} \ldots\ldots (94a)$$

or

$$M_{AB} = M_{FAB} - R_{AB}\,M_{FAB} - F_{BA}\,R_{BA}\,M_{FBA} \ldots\ldots (94b)$$

Eq. 94*a* has the advantage of requiring the addition of only two terms. However, slide rule work is simplified in Eq. 94*b*, since the CI-scale may be used to obtain the second term of the equation for M_{AB}, and the third term of the equation for M_{BA} may then usually be found with either the C-scale or the CF-scale without changing the position of the sliding scale.

TABLE 22.—MODIFIED SOLUTION OF EXAMPLE 1(*a*)



Line	Quantity	A	B	B	C	C	D	D	E
1	F		0.50	0.267	0.286	0.250		0	
2	J		1.333	1.156	1.167	1.143		1.000	
3	R			0.464	0.505				
4	M_F			+12	−12				
5	$(1 - R)\, M_F$			+6.43	−5.94				
6	$-F\, R\, M_F$			+1.74	−1.49				
7	M	−4.08	−8.17	+8.17	−7.43	+7.43	+1.86	−1.86	0

Table 22 shows the solution of Example 1(*a*) by Eqs. 91, 92, and 94*a*. Since the I/L-ratios are the same for all members, the value $K = 1$ was chosen for simplicity. The computations start by entering in line 1 the known value $F_{BA} = 0.5$. (In a strict sense, the computations start with the known joint stiffness $J_A = \infty$. Eq. 91 gives $F = 0.5$ when $J \rightarrow \infty$.) With F_{BA} known, the value of J_{BA} is calculated from Eq. 92 and entered in line 2, Table 22. With J_{BA} known, F_{CB} is determined from Eq. 91 and recorded in its proper place. Calculations to the right stop with J_{CB} since the DC-factors are not needed for this example. The known value $F_{DE} = 0$ is recorded next, and computations to the left proceed as outlined, stopping with the factors for span BC. It will be observed that the additions in Eqs. 91 and 92 can be performed mentally, and the equations are easily remembered after using them a few times.

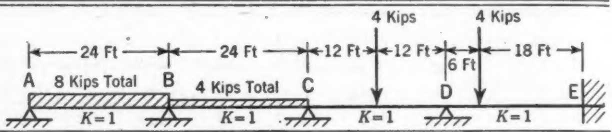
Only two values of R are needed for this example, and they are computed and recorded in line 3, Table 22. The remainder of the calculations is self-explanatory. However, it should be noted that the second term of Eq. 94*a* is a "carry-over" and must be so entered in the table.

The example in Table 23 is given to show the procedure when more than one span is loaded. In this example, Eq. 94*b* is used. It is unnecessary to calculate values of J at the end supports, since $R = 0$ for a fixed-end support and $R = 1$ for a hinged-end support. After the values of R and M_F are determined, $M_{FAB} = 16$ is multiplied by the R-value immediately above, and the negative of the result recorded immediately below in line 5. This product is then multiplied by F_{AB} and the result, -4.29, recorded in line 6 at point B. After lines 5 and 6 have been completed, line 7 is found by adding lines 4, 5, and 6.

The first step in calculating carry-overs to the right is to record at point B of span BC in line 8 the balancing moment for $M_{BA} = -12.86$. The carry-

over +3.46 is then recorded at point C. This value added to the moment immediately above gives the balancing monent (+1.74) to be recorded at point C of span CD. The carry-over to point D is then computed, and the process continued until point E is reached. Line 9 is computed similarly, and the final moments in line 10 are determined by adding rows 7, 8, and 9.

TABLE 23.—ILLUSTRATIVE EXAMPLE; MORE THAN ONE SPAN LOADED



Line	Quantity								
1	F	0.268	0	0.269	0.250	0.286	0.267	0.500	0.268
2	J		1.000	1.156	1.143	1.167	1.156	1.333	
3	R	1.000	0.464	0.536	0.495	0.505	0.464	0.536	0
4	M_F	+16	−16	+8	−8	+12	−12	+13.5	−4.5
5	$-R\,M_F$	−16	+7.43	−4.29	+3.96	−6.06	+5.57	−7.24	0
6	$-F\,R\,M_F$	0	−4.29	+0.99	−1.16	+1.49	−1.73	0	−3.62
7	M	0	−12.86	+4.70	−5.20	+7.43	−8.16	+6.26	−8.12
8	Right[a]			+12.86	+3.46	+1.74	+0.50	+7.66	+3.83
9	Left[a]		−3.26	−1.44	−5.76	−1.67	−6.26		
10	M	0	−16.12	+16.12	−7.50	+7.50	−13.92	+13.92	−4.29

[a] Carry-over to the right or left.

The tabular solution can be extended to frames of the type shown in Fig. 8. For sidesway calculations, Eqs. 20 can be expressed in terms of the factors F, J, and R. Consider the leg CD of the frame of Fig. 8. From Eq. 20*a*,

$$M_{CD} = \frac{3\,E\,I\,\Delta}{2\,L}\left[\frac{2}{L_{DC} - \dfrac{L^2}{4\,L_{CD}}} + \frac{L/L_{DC}}{L_{CD} - \dfrac{L^2}{4\,L_{DC}}}\right] \quad \text{.......(95a)}$$

Substituting from Eq. 93*g*, with proper changes in notation, Eq. 95*a* becomes, for M_{AB},

$$M_{AB} = \frac{\Delta}{L}\left[J_{AB}\,(1 - R_{AB}) + F_{BA}\,J_{BA}\,(1 - R_{BA})\right] \quad \text{.......(95b)}$$

The solution of the frame of Fig. 8, with sidesway restrained, is given in Table 24. As in the previous examples in this discussion, relative values of K are used. The computation starts with the known values F_{AB}, F_{CD}, F_{EF}, etc., from which the values of J_{AB}, J_{CD}, J_{EF}, etc., are computed by Eq. 92. The values of F_{CA} and J_{CA} are determined next, using the known value of F_{AB}. In calculating F_{EC}, it must be remembered that the stiffness factor J in Eq. 91 is given by Eq. 13*b* as the sum of J_{CA} and J_{CD}—that is,

$$F_{EC} = \tfrac{1}{2}\,\frac{J_{CA} + J_{CD}}{J_{CA} + J_{CD} + K_{EC}} \quad \text{..................(96)}$$

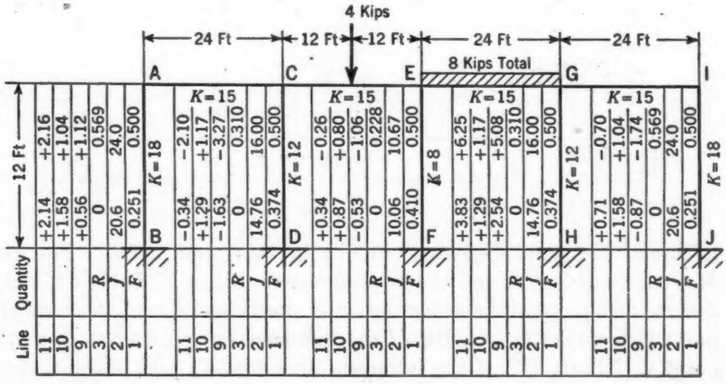
Calculations proceed in this manner until the JI-values have been determined. The process must then be repeated, starting with F_{IJ} and working to the left until joint B is reached. Because of the symmetry of the frame in this example,

values from point J to the left are the same as those from point B to the right. Finally, values of F and J for joints D, F, and H are determined.

The first step in determining line 8 is the distribution of M_{EC} to members EF and EG in proportion to their stiffness factors (J_E). The carry-over from point E to point G is calculated next and the final moment $M_{GE} = -10.71$ is distributed. Carry-over and distribution are continued until joint J is

TABLE 24.—MODIFIED SOLUTION OF EXAMPLE 3

Line	Quantity	A		C.		E		G	I
11	Final moment	−2.16	−4.38	+6.49	−16.48	+16.74	−11.13	+4.89	+0.70
10	Correction for sidesway	−1.04	−0.74	−0.42	−0.40	−0.40	−0.42	−0.74	−1.04
9	Final moment; no sidesway	−1.12	−3.64	+6.91	−16.08	+17.14	−10.71	+5.63	+1.74
8	Distribution and carry-over			−2.54	−7.34	+.550	+1.90		
7	M			+9.45	−8.74	+11.64	−12.61		
6	$-F\,R\,M_F$			+1.60	−1.37	+1.82	−2.14		
5	$-R\,M_F$			−4.15	+4.63	−6.18	+5.53		
4	M_F			+12	−12	+16	−16		
3	R	0.431	0.344	0.346	0.386	0.386	0.346	0.344	0.431
2	J	18.15	17.73	17.95	18.14	18.14	17.95	17.73	18.15
1	F	0.347	0.308	0.329	0.346	0.346	0.329	0.308	0.347



reached. Carry-over and distribution to the left starts with M_{EG}. Line 8 is omitted for the intermediate columns since the final moments can be determined by equilibrium of joints C, E, and G.

Because of symmetry, only half of the frame is shown in Fig. 31. All distributions in line 4 are completed before starting line 5. The first entry in line 5 is the carry-over to point C from point A. This carry-over is distributed, giving $M_{CE} = +2.85$. The carry-over to point E is based on the sum of the moments +2.85 and −8.33 at point C. The carry-over from point E to point F is recorded in line 6 at point C in span CE. The process continues until joint B is reached. As in Table 24, no distributions are recorded for the intermediate columns. The moment M_{CD} in line 9 is calculated from equilibrium of joint C, and the total distribution, +3.37, is the difference between

lines 3 and 9. The carry-over to point D can then be found, and the final moment M_{DC} determined. No explanation of the final steps, lines 10 and 11, Table 24, is necessary.

The writer has found the procedure described especially simple and easily remembered for continuous beams. It can be used very effectively in the construction of influence lines, and the advantage of basing the procedure on

7	Moment with sidesway	−17.78	−12.70	−7.23	−6.85	−6.85
6	Carry-over to left	− 2.26	+ 0.92	−1.75	+1.13	−1.80
5	Carry-over to right		− 5.39	+2.85	−1.80	+1.13
4	Distribution	−15.52	− 8.23	−8.33	−6.18	−6.18

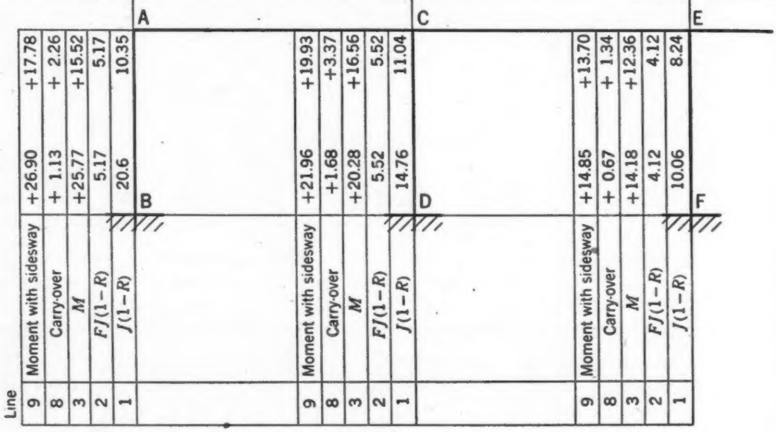


FIG. 31

fixed-end moments becomes evident in this application. For example, the M_F-values of line 4, and the final moments of line 7, Table 22, can be quickly determined for a sufficient number of positions of load between point B and point C to plot the part BC of the influence lines for moment at point B and point C. Parts BC of the influence lines for M_A and M_D can then be found by multiplying ordinates of the M_B and M_C influence lines by F_{BA} and F_{CD}, respectively. As an alternative procedure, the equations of the influence lines for M_B and M_C may be found. For example:

$$M_{FBC} = 24\,k\,(1 - k)^2 \qquad (97a)$$

and

$$M_{FCB} = 24\,k^2\,(1 - k) \qquad (97b)$$

in which $k = x/L$, and x is the abscissa of the unit load position measured from point B. Following the calculations of lines 5, 6, and 7, Table 22, the final moments are

$$M_{BC} = 9.46\,k\,(1 - k)\,(1.36 - k) \qquad (98a)$$

and

$$M_{CB} = 8.45\,k\,(1 - k)\,(0.406 + k) \qquad (98b)$$

From Eqs. 98 any desired number of influence line ordinates can be easily calculated. The maximum ordinate of the BC-segment can also be determined readily. Equating to zero the first derivatives with respect to k, it is found that $k = 0.381$ and $k = 0.616$ for maximum values of M_{BC} and M_{CB}, respectively. It should be observed that this also determines maximum ordinates of the BC-segment of the influence lines for M_A and M_D, since these moments are found by multiplying M_B and M_C by the proper carry-over factors, as explained previously.

The practicing engineer prefers (and rightly so) an easily remembered, compact analysis, with a minimum number of formulas. In this discussion an attempt has been made to meet these objections to the authors' procedure. It is quite certain that most engineers will still prefer the Hardy Cross method because of its simplicity. The writer believes, however, that the application of the method to the construction of influence lines offers definite advantages.

William A. Conwell,[58] M. ASCE.—There is something of analytical beauty in the concept which the authors present in this paper. They analyze frames as they are—that is, as structures with joints, the rotation of each of which is proportional to the moments impressed upon it. Many of those who

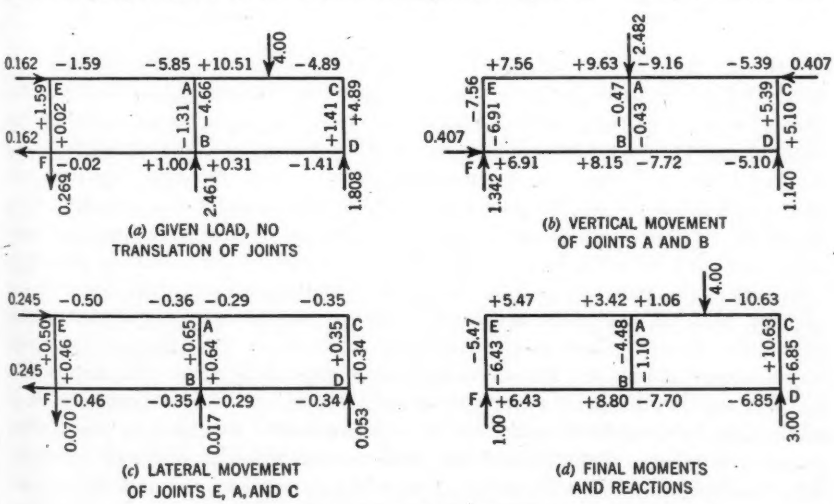


Fig. 32.—Steps in Solution of Moments and Reactions of Example 5 by Moment Distribution

are daily engaged in analysis of continuous frames will appreciate this sophisticated outlook and will, henceforth, be inclined to treat as naive any method which does not treat structures as they actually are.

Although the proposed method of solution involves thought that is closer to the actual condition of the structure than do accepted methods, it also contains formulas which must be remembered to be applied and entails compu-

[58] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

tation which cannot be wholly written on a sketch of the structure. These elements divert the mind of the designer from the structure and are, of course, ones which moment distribution[8] was to eliminate.

It is believed that the relative merits of the methods of elastic supports and of moment distribution can best be compared by an application to a particular problem. With the thought of providing such a pointed comparison, the writer has solved Example 5 by moment distribution and gives the results of this analysis in Fig. 32. These values were obtained after an average of seven cycles of moment distribution, which is not a great chore. It was necessary, of course, to solve two equations simultaneously to determine the proportion of the final moments resulting from sidesway and the vertical movement of joints A and B. Aside from this disgression, it was possible to write all computations on a sketch of the frame. Assessment of the relative value of the methods can then be made by comparison of Fig. 32 with seven cycles of moment distribution visualized thereon and Tables 8 and 9 with their supplementary computations.

In the final analysis, the value of the method of elastic supports will be determined in design offices where it will meet competition with other present and proposed methods. One's first impression, however, is that the authors' presentation is one for the expert in continuous frame analysis, rather than for the engineer who meets such design problems only occasionally.

HARRY GARFINKEL,[59] ESQ.—For many years the writer has examined with unabated interest the methods proposed by various investigators for the solution of stresses in continuous frames. All have concentrated on reducing the time element; many have accomplished this purpose remarkably well for the simpler types of frame. In the more complex type of structure, however, the amount of work still required for solution of an actual engineering design leaves much to be desired, both as to method and clarity. Computations for such structures can be quite lengthy and must be reviewed for checking purposes. Complex sign conventions and numerous distributions required for varying loading patterns can cause great annoyance unless the method of analysis is direct and avoids unnecessary artifices for a solution. It is therefore of great importance that any new method should avoid these difficulties as far as possible.

The authors propose a method of analysis of continuous frames that is claimed to be "classical" and "exact." Presumably, these terms are applied because the basic equations of the method originate in classical concepts. The authors' method and steps of operation, however, cannot be termed "exact" in the same sense that such a term is applied to methods, based on least work, slope deflection, or virtual work, which are direct and lead to exact results. Inherent in the authors' method is the artifice of fixing and relaxing one or more joints of the frame. Simultaneous equations are then required to find the releasing moments, which must additionally be distributed to the

[8] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

[59] Civ. Engr., Board of Transportation, City of New York, New York, N. Y. (Mr. Garfinkel died on September 17, 1947, and this discussion was prepared for publication by Nathan D. Brodkin, Assoc. M. ASCE.)

frame, resulting in little if any saving in time or energy over presently known moment-distribution methods. The authors have attempted the solution of the problem by first calculating approximate values of the frame invariants and then, with the aid of these, distributing moments caused by the loading.

Criticism must be leveled at the manner in which the authors determine these invariants. Introduction of the elastic length concept needlessly complicates the formulas and results in no actual benefit. In re-entrant or completely enclosed frames, such as Vierendeel trusses, the problem of finding the approximate carry-over factors or transmission coefficients by temporarily fixing joints against rotation is unnecessary and entails needless work in distributing moments caused by releasing the joints.

In laying the theoretical basis for their method, the authors have not carried their work through to the logical conclusion of finding all the invariants of the frame. Since these quantities are independent of the manner of loading, they can be determined completely and exactly for any given frame, and it is not necessary to resort to fixing one or more joints against rotation.

The foregoing problem has been completely solved for uniform, nonuniform, and curved members by R. C. Brumfield, M. ASCE, who has shown[56,60] how the complete invariants for a frame, whether re-entrant or not, may be obtained quickly, exactly, and simply, and then with the aid of these, how to distribute in one cycle the moments caused by the loads.

The three major invariants of a continuous frame are:

1. A transmission coefficient (carry-over factor), which is that fraction of the moment arriving at one end of a beam as a consequence of a unit moment applied at the other end (every member having two transmission coefficients);
2. A moment split, which is that fraction by which moment arriving at a joint through one of the radiating members of the joint is to be multiplied to give the induced moment in the member to which the moment split applies; and
3. A torque split, which is that fraction by which torque applied to a joint is to be multiplied to give the induced torque (or moment) in the member to which it applies.

Once the exact transmission coefficients have been determined, the bending moments at the ends of a loaded member may be found directly in terms of the fixed-end moments and the transmission coefficients.[61]

Comparisons are invidious, and particular problems may be solved more readily by one method than by another. Nevertheless, the writer would like to indicate how simply the invariants, especially those for closed or Vierendeel frames, may be found without arbitrarily fixing any joints, and then with the aid of the three major invariants to complete the solution of moment distribution for Example 6 as a comparison with the authors' method.

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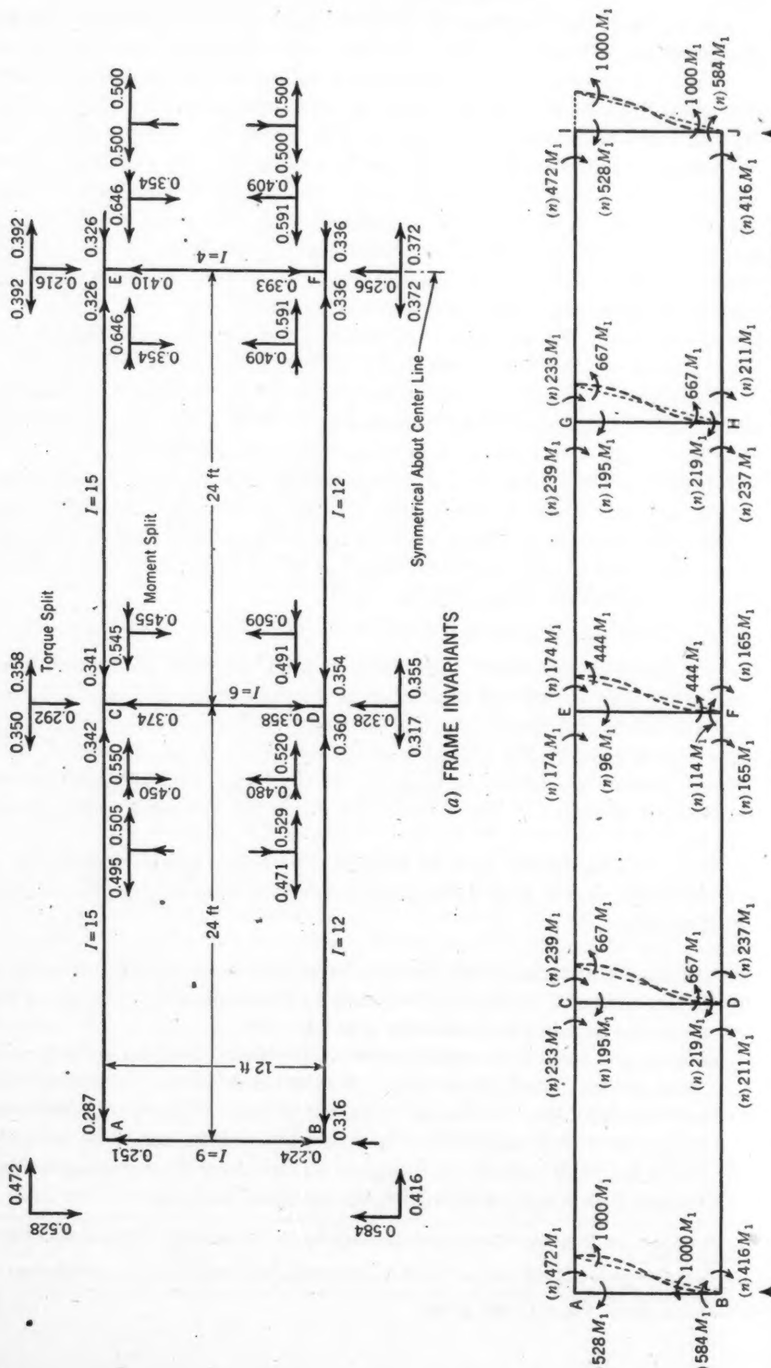
56 "Moving Loads on Restrained Beams and Frames," by R. C. Brumfield, *Transactions*, ASCE, Vol. 111, 1946, p. 805.

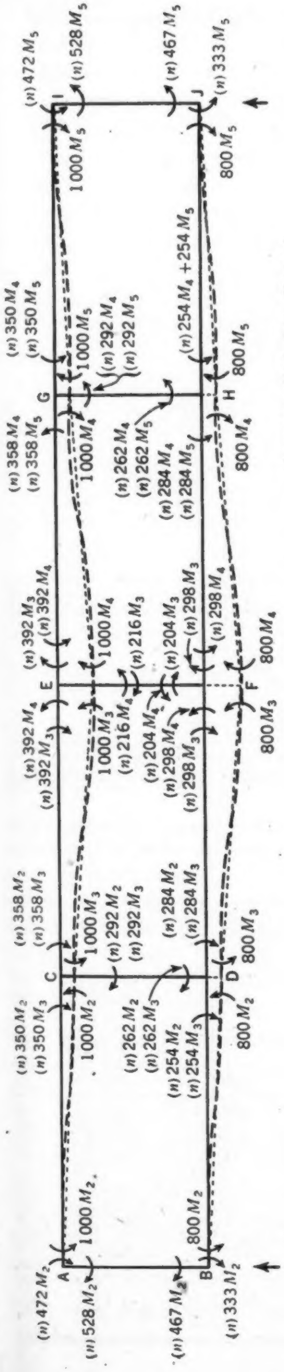
60 "Restrained Beams and Rigid Frames," by R. C. Brumfield, 1941 (printed in mimeograph form by The Cooper Union, New York, N. Y.).

61 *Transactions*, ASCE, Vol. 111, 1946, p. 838.



(a) FRAME INVARIANTS

(b) HORIZONTAL SWAY



(c) VERTICAL SWAY

Note: (n) = Neutralizing Moment

FIG. 33.—ARTIFICIAL AND NEUTRALIZING MOMENTS FOR SWAY IN FRAME OF EXAMPLE 6

The invariants are defined as follows:

$$K = \frac{I}{L} \quad \ldots\ldots\ldots\ldots (99)$$

C_{AB} and C_{BA} are transmission coefficients from A toward B and vice versa; m_{AB} and t_{AB} are the moment split and torque split, respectively, for member AB at joint A; i', the stiffness index, is a measure of the resistance which a member, together with the restraints applied to it at one end, offers to the rotation of the joint to which it attaches at the opposite end, or

$$i' = \frac{K}{2 - C} \quad \ldots\ldots\ldots (100)$$

R', the restraint index, is a measure of the resistance to rotation offered to a member, when it is used to rotate a joint, by the other members radiating from the joint, or

$$R' = i'_1 + i'_2 + i'_3 + \cdots + i'_n \ldots (101)$$

and R'_T, the restraint index in torsion, is a measure of the resistance to rotation offered to a member by all the members radiating from a joint, or

$$R'_T = i'_0 + i'_1 + i'_2 + i'_3 + \cdots + i'_n \ldots (102)$$

The primes used in Eqs. 100, 101, and 102 indicate the reciprocals of the values introduced by Professor Brumfield.[56,60] For beams of uniform section, the value of the transmission coefficient is given by

$$C = \frac{1}{2 + \frac{K}{R'}} \quad \ldots\ldots\ldots\ldots (103)$$

for moment split, by

$$m_n = \frac{i'_n}{R'} \quad \ldots\ldots\ldots\ldots (104)$$

and for torque split, by

$$t_n = \frac{i'_n}{R'_T} \quad \ldots\ldots\ldots\ldots (105)$$

Calculation of the invariants is a slide rule computation and may be kept in

TABLE 25.—FRAME INVARIANTS (CHARACTERISTICS) FOR EXAMPLE 6 (FIG. 12)[a]

Joint	Member	Moment of inertia, I (in.^{4})	Length, L (ft)	K $\left(=\frac{I}{L}\right)$	$2-C$	i' $\left(=\frac{K}{2-C}\right)$	R'	Coefficient: Symbol	Coefficient: Value	Moment Split			Torque split
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
A	AB	9	12	0.750	1.776	0.422	0.377	C_{BA}	0.251	BA	1	....	0.528
	AC	15	24	0.625	1.659[b]	0.377	0.422	C_{CA}	0.287	1	CA	....	0.472
						0.799[c]							
C	CD	6	12	0.500	1.642	0.305	0.738	C_{DC}	0.374	DC	0.450	0.455	0.292
	CA	15	24	0.625	1.713	0.365	0.678	C_{AC}	0.342	0.495	AC	0.545	0.350
	CE	15	24	0.625	1.674[b]	0.373	0.670	C_{EC}	0.341	0.505	0.550	EC	0.358
						1.043[c]							
E	EF	4	12	0.333	1.607	0.207	0.754	C_{FE}	0.410	FE	0.354	0.354	0.216
	EC	15	24	0.625	1.659	0.377	0.584	C_{CE}	0.326	0.500	CE	0.646	0.392
	EG	15	24	0.625	1.659	0.377	0.584	C_{GE}	0.326	0.500	0.646	GE	0.392
						0.961[c]							
F	FD	12	24	0.500	1.646	0.304	0.514	C_{DF}	0.336	DF	0.500	0.591	0.372
	FE	4	12	0.333	1.590	0.210	0.608	C_{EF}	0.393	0.409	EF	0.409	0.256
	FH	12	24	0.500	1.646	0.304	0.514	C_{HF}	0.336	0.591	0.500	HF	0.372
						0.818[c]							
D	DB	12	24	0.500	1.684	0.297	0.641	C_{BD}	0.360	BD	0.471	0.491	0.317
	DC	6	12	0.500	1.626	0.308	0.630	C_{CD}	0.358	0.480	CD	0.509	0.328
	DF	12	24	0.500	1.666[b]	0.333	0.605	C_{FD}	0.354	0.520	0.529	FD	0.355
						0.938[c]							
B	BA	9	12	0.750	1.749	0.429	0.304	C_{AB}	0.224	AB	1	....	0.584
	BD	12	24	0.500	1.646[b]	0.304	0.429	C_{DB}	0.316	1	DB	....	0.416
						0.733[c]							

[a] Also Fig. 33(a). [b] Estimated value. [c] Value of R'_T for respective joint.

TABLE 26.—TABULATION AND DISTRIBUTION

Line	Symbol	Description	Joint A: ← M_{AB} →		← M_{AC} →		Joint C: ← M_{CA} →		← M_{CD} →		← M_{CE} →	
(1)	(2)	(3)	(4)		(5)		(6)		(7)		(8)	
1	$\left\{\begin{matrix}M_{CE}\\M_{EC}\end{matrix}\right\}$	Primary distribution	1.46	0.06	0.06	1.46		5.10	0.04	4.24	9.32	0.02
2	$\left\{\begin{matrix}M_{EG}\\M_{GE}\end{matrix}\right\}$	Primary distribution		0.40	0.40		1.41		1.16	0.05	0.03	2.55
3	ΣM_0	No joint translation	1.00			1.00		3.69		3.09	6.78	
4	ΣM_1	Artificial moments plus distributed neutralizing moments	392			392		298	491			193
5	ΣM_2			540	540		565			317		248
6	ΣM_3		103			103		250		314	564	
7	ΣM_4			5	5			48		58	106	
8	ΣM_5			4	4		5		11			16
9	M_1	Line 4 × − 0.002423		0.95	0.95		0.72			1.19	0.47	
10	M_2	Line 5 × 0.081047		43.77	43.77		45.79			25.69		20.10
11	M_3	Line 6 × 0.057996	5.97			5.97		14.50		18.21	32.71	
12	M_4	Line 7 × 0.045476		0.23	0.23			2.18		2.64	4.82	
13	M_5	Line 8 × 0.093567		0.37	0.37		0.47		1.03			1.50
14	ΣM	Final moment (line 3 + lines 9 to 13)		38.35	38.35		26.61			49.79	23.18	

tabular form. For Example 6, the invariants are as shown in Table 25. For convenience the invariants are also placed on the frame diagram in Fig. 33(a).

If the concept of right and left arrows as shown in Table 26 is used, the chances for errors in sign are obviated. Normal bending moment, indicated by an arrow to the right, is clockwise moment applied to the beam when taken as a free body. Reverse bending moment, indicated by an arrow to the left, is a counterclockwise moment. For a downward load the moment at the right end is normal and at the left end, reverse. The moments are distributed to the frame in accordance with the frame invariants. After going through a joint, the moment changes in character. When going from one end of a beam to the other, moment remains the same in character. If only two members radiate from a joint, the moment split at that joint is unity; if more than two members occur at a joint, the moment splits would, of course, be fractional. In order to determine the transmission coefficients, it is not necessary to assume fixed joints as the authors have done. Much simpler is the method of estimating one of the coefficients and using it to compute another, which, as a result of the rapidly converging nature of Eq. 103, will be correct within one or two thousandths.

It is known that, for an elastically restrained member of uniform section, the transmission coefficient lies between 0 and 0.500, for free ends and fixed ends, respectively. For a series of equal spans having equal moments of inertia, it can be shown[62] that $C = 0.268$. Eq. 103 may be written in the form,

$$C_{AB} = \frac{1}{2 + \dfrac{K_{AB}}{R'_{AB}}} \qquad \text{(106)}$$

[62] "Restrained Beams and Rigid Frames," by R. C. Brumfield, 1941 (printed in mimeograph form by The Cooper Union, New York, N. Y.), p. 9.

OF MOMENTS, EXAMPLE 6 (FIG. 33)

JOINT E						JOINT G						JOINT I			
← M_{EC} (9)	→	← M_{EF} (10)	→	← M_{EG} (11)	→	← M_{GE} (12)	→	← M_{GH} (13)	→	← M_{GI} (14)	→	← M_{IG} (15)	→	← M_{IJ} (16)	→
0.02	8.68	3.07	0.04	5.63		1.91	0.02	0.04	0.87		1.06		0.30	0.30	
	7.51	0.06	4.10	11.58	0.03	0.03	12.43	5.66	0.05	6.79		1.94	0.08	0.08	1.94
	16.17		1.01	17.18			10.51	4.78		5.73		1.56			1.56
	178	356			178		193	491			298		392	392	
	87	36		51		16			11		5		4	4	
542			251		291		106	58		48			5	5	
291		251			542		564	314		250		103			103
	51		36	87		248		317			565		540	540	
0.43			0.86	0.43		0.47			1.19	0.72		0.95			0.95
	7.05	2.92		4.13		1.30			0.89		0.41		0.32	0.32	
31.43			14.56		16.88		6.15	3.36		2.79			0.29	0.29	
13.23		11.41			24.65		25.65	14.28		11.37		4.68			4.68
	4.77		3.37	8.14		23.20		29.66			52.86		50.53	50.53	
17.10			5.47		11.65		17.34	50.00			32.66		43.95	43.95	

TABLE 26.—

Line	Symbol	Joint B				Joint D							
		← M_{BA} →		← M_{BD} →		← M_{DB} →		← M_{DC} →		← M_{DF} →		← M_{FD} →	
(1)	(2)	(17)		(18)		(19)		(20)		(21)		(22)	
1	$\begin{Bmatrix} M_{CB} \\ M_{BC} \end{Bmatrix}$	0.33	0.26	0.26	0.33	0.82	0.12	17	1.52	0.86	0.21	0.27	0.63
2	$\begin{Bmatrix} M_{BG} \\ M_{GB} \end{Bmatrix}$	0.10	0.09	0.09	0.10		0.37	0.42	0.18	0.35	0.22	1.01	0.07
3	ΣM_0	0.08			0.08	0.33			1.11	0.78		0.58	
4	ΣM_1	362			362		281	480			199		175
5	ΣM_2		497	497		502			301		201		72
6	ΣM_3	74			74		177		298	475		457	
7	ΣM_4	7			7		31		55	86		218	
8	ΣM_5		3	3		3		8			11		35
9	M_1		0.88	0.88		0.68			1.16	0.48		0.42	
10	M_2		40.28	40.28		40.69			24.40		16.29		5.84
11	M_3	4.29			4.29		10.27		17.28	27.55		26.50	
12	M_4	0.32			0.32		1.41		2.50	3.91		9.91	
13	M_5		0.28	0.28		0.28		0.75			1.03		3.28
14	ΣM		36.75	36.75		30.30			45.70	15.40		28.29	

and since

$$R'_{AB} = i'_{BD} = \frac{K_{BD}}{2 - C_{BD}} \qquad (107)$$

for the two member joint, then

$$C_{AB} = \frac{1}{2 + \dfrac{K_{AB}}{\left(\dfrac{K_{BD}}{2 - C_{BD}}\right)}} \qquad (108)$$

which is the form used in estimating C_{AB}. If member BD connects into a stiffer member, the coefficient will be greater than 0.268, and conversely.

Although the Vierendeel truss of Example 6 is symmetrical in form, it is unsymmetrically loaded. To estimate certain of the transmission coefficients (carry-over factors) of a re-entrant frame of this type, an average value must be estimated for $i' = \sum \frac{K_n}{2 - C_n}$ for each of the members radiating from the joint, except the member for which the transmission coefficient is being estimated. For example, to estimate C_{AC} (Fig. 33(a)), $C_{CE} > 0.268$, say, 0.300, and $C_{CD} > 0.268$, say, 0.350. Then,

$$C_{AC} = \frac{1}{2 + \dfrac{K}{R'}} = \frac{1}{2 + \dfrac{K_{AC}}{\dfrac{K_{CE}}{2 - C_{CE}} + \dfrac{K_{CD}}{2 - C_{CD}}}} = \frac{1}{2 + \dfrac{0.625}{\dfrac{0.625}{2 - 0.300} + \dfrac{0.500}{2 - 0.350}}}$$

$= 0.341$. If an average value for $2 - C_n$ is assumed, the general formula becomes

$$C_{AC} = \frac{1}{2 + \dfrac{K\,(2 - C_{avg})}{K_1 + K_2 + K_3 + \cdots + K_n}} \qquad (109)$$

(*Continued*)

JOINT F				JOINT H						JOINT J			
← M_{FE} → (23)		← M_{FH} → (24)		← M_{HF} → (25)		← M_{HG} → (26)		← M_{HJ} → (27)		← M_{JH} → (28)		← M_{JI} → (29)	
1.20	0.13	0.05	0.76	0.16	0.27	0.14	0.31	0.28		0.08	0.07	0.07	0.08
0.18	1.60	0.84	0.36	0.28	1.15	2.03	0.21	0.15	1.10	0.43	0.34	0.34	0.43
	0.35		0.23		0.98	1.65			0.67	0.10			0.10
350			175		199	480			281		362	362	
37		35		11			8		3		3	3	
	239		218		86	55		31		7			7
239			457		475	298		177		74			74
	37	72		201		301			502		497	497	
	0.85	0.43		0.48			1.16	0.68		0.88			0.88
3.00		2.84		0.89			0.65		0.24		0.24	0.24	
	13.86		12.64		4.99	3.19		1.80		0.41			0.41
10.87			20.78		21.60	13.55		8.05		3.37			3.37
	3.46	6.74		18.81		28.16			46.97		46.50	46.50	
	4.65		23.64		7.39	44.74			37.35		41.99	41.99	

The value of C_{avg} is not critical. Thus, for $C_{avg} = 0.250$, $C_{AC} = 0.336$; for $C_{avg} = 0.300$, $C_{AC} + 0.340$; and for $C_{avg} = 0.350$, $C_{AC} = 0.343$. An average value of C_n based on a general mental estimate of the coefficients, in this case 0.350, is used. Therefore, $C_{CE} = \dfrac{1}{2 + \dfrac{1.650 \times 0.625}{0.625 + 0.333}} = 0.326.$ Similarly, $C_{BD} = 0.354$ and $C_{DF} = 0.334$. These coefficients give starting points, so that Table 25 becomes routine procedure, advantage being taken of frame symmetry.

The primary bending moments for the given loading without the effects of sway or vertical displacements are obtained from the following equations:[61]

$$M_R = A_R\,[(2\,M_{FR} + M_{FL}) - C_L\,(2\,M_{FL} + M_{FR})] \quad \text{.....}(110a)$$

$$M_L = A_L\,[(2\,M_{FL} + M_{FR}) - C_R\,(2\,M_{FR} + M_{FL})] \quad \text{.....}(110b)$$

$$A_R = \frac{C_R}{1 - C_L\,C_R} \quad \text{....................}(111a)$$

and

$$A_L = \frac{C_L}{1 - C_L\,C_R} \quad \text{....................}(111b)$$

in which C_R and C_L are the transmission coefficients; M_{FR} and M_{FL} are the fixed-end moments; and M_R and M_L are the restraining moments at the right and left beam ends, respectively.

For span CE: $C_R = 0.326$, $C_L = 0.341$, $A_R = 0.366$, $A_L = 0.384$, $M_{EC} = M_R = 0.366 \times 0.659 \times \frac{3}{8} \times 4 \times 24 = -8.68$ ft-kips, and $M_{CE} = M_L = 0.384 \times 0.674 \times \frac{3}{8} \times 4 \times 24 = -9.32$ ft-kips. For span EG: $C_R = 0.341$, $C_L = 0.326$, $A_R = 0.384$, $A_L = 0.366$, $M_{GE} = M_R = -0.384 \times 0.674 \times \frac{1}{2}$

$\times\ 8 - 24 = -\ 12.43$ ft-kips, and $M_{EG} = M_L = -\ 0.366 \times 0.659 \times \frac{1}{2} \times 8 \times 24 = -\ 11.58$ ft-kips. These primary moments have been distributed in Table 26, lines 1 and 2, and totaled in line 3. Moments have been distributed through three joints from the joint of origin.

The moments caused by the horizontal sway and vertical displacements must now be determined. Since dissymmetry of loading exists, the frame will tend to lurch sideway. It is allowed to sway laterally and the artificial moments required to rotate the tangents at the column ends parallel to their original direction are shown in Fig. 33(*b*). Neutralizing moments at each joint necessary to balance the artificial moments are apportioned among the radiating members in accordance with the joint torque split obtained from Fig. 33(*a*), or Table 25, and are designated (*n*) in Fig. 33(*b*). In a similar manner, artificial and neutralizing moments are found for vertical displacement as shown in Fig. 33(*c*). The neutralizing moments are distributed to the frame in accordance with the frame invariants given in Table 25 and Fig. 33(*a*). The summations of the distributed neutralizing moments, together with the artificial moments, are given in lines 4 to 8, Table 26. Since there are five unknown moments of translation, five equations must be written and solved simultaneously. The coefficients of M_0 to M_5, forming these equations, are obtained from Table 26 as follows:

For the first equation, the sum of the horizontal shears in the posts AB, CD, EF, GH, and IJ in terms of M_0 to M_5 is set equal to zero. Designating normal moments as plus and reverse moments as minus,

$$-\ 4{,}156\ M_1 + 1{,}594\ M_2 + 814\ M_3 - 814\ M_4 - 1{,}594\ M_5 - 0.30 = 0 \quad \text{(112)}$$

For the second equation, the internal shear at A and B in members AC and BD is set equal to the left reaction, $R_L = 5.5$ kips, so that

$$+\ 1{,}333\ M_1 - 2{,}104\ M_2 + 604\ M_3 + 81\ M_4 - 15\ M_5 + 4.44 = -\ 5.5 \times 24 \quad \text{(113)}$$

For the third equation, the internal shear to the right of C and D in members CE and DF is set equal to the left reaction. Thus,

$$+\ 745\ M_1 + 608\ M_2 - 2{,}038\ M_3 - 701\ M_4 + 113\ M_5 + 8.03 = (-\ 5.5 + 2)\ 24 \quad \text{(114)}$$

For the fourth equation, the internal shear at I and J in members GI and HJ is set equal to the right reaction, $R_R = 6.5$ kips, giving

$$+\ 1{,}333\ M_1 + 15\ M_2 - 81\ M_3 - 604\ M_4 + 2{,}104\ M_5 - 6.71 = 5 \times 24 \quad \text{(115)}$$

The fifth equation is obtained from the vertical displacement diagram (Fig. 33(*c*)). The sum of the deflections of AC and CE must be equal to the sum of the deflections of IG and GE, which can be expressed in terms of M_2 to M_5. Thus,

$$M_2 + M_3 = M_4 + M_5 \quad \text{(116)}$$

Solving Eqs. 112 to 116, simultaneously, gives $M_1 = -\ 0.002423$, $M_2 = +\ 0.081047$, $M_3 = +\ 0.057996$, $M_4 = +\ 0.045476$, and $M_5 = +\ 0.093567$.

In step (*h*) of Example 6, the authors have set up four equations for vertical shear correction factors. Nowhere, however, does a similar equation or correction factor appear for the horizontal shear or horizontal movement of the top chord. The effects of both horizontal sway and vertical displacement of the joints must be considered as simultaneous actions in determining correction factors, since these factors for vertical and horizontal shear are interrelated. Such correction (for horizontal sway) is not obvious from the authors' calculations in Examples 4, 5, and 6.

The values of M_1 to M_5 found from Eqs. 112 to 116 are multiplied by the respective coefficients in lines 4 to 8, Table 26, to give lines 9 to 13, and are added to the primary moments due to loading in line 3 to give the final moments in line 14. These are to be compared with the authors' values (line 25, Table 13).

In the "Summary" the authors claim their method to be particularly adapted to the solution of Vierendeel trusses. Nevertheless, they have carried the solution of the truss of Example 6 only to the point of finding primary moments corrected for joint translation, despite the fact that a Vierendeel truss of several panels will develop secondary moments due to changes in member lengths caused by the axial stresses. These secondary moments are considerable and cannot be ignored. When the calculations for secondary moments are added to the work required by the authors' method for finding primary moments, it becomes doubtful whether any advantage is to be gained by adopting the proposed system of analysis.

It has been shown elsewhere[63] how the method as used by the writer to find the primary moments has been extended to solve the secondary stresses for this type of truss.

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[63] *Transactions*, ASCE, Vol. 111, 1946, p. 848.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

REFLECTIONS ON STANDARD SPECIFICATIONS FOR STRUCTURAL DESIGN

Discussion

BY A. M. FREUDENTHAL

A. M. FREUDENTHAL,[14] M. ASCE.—In discussing a previous paper by the writer,[2] to which the present paper might be considered a sequel, one of the participants pointed to the surprising lack of interest among engineers in the fundamentals of their profession. That there appears to be no reason to revise this statement is obvious by the reluctance to discuss, and thus help to solve, one of the most fundamental problems facing the structural designer—that of the design specification.

This classification of the problem is hardly an exaggeration; there is no single problem of greater importance or of greater consequence in the daily work of the individual structural designer than that of being able to work on the basis of rational and sound design specifications, which are devised to help him to use the material at his disposal with the maximum efficiency consistent with adequate safety, without, at the same time, affecting the expediency of his design procedure by the introduction of cumbersome and time consuming methods of analysis. The average designer, however, is ready to discuss in greatest detail, and apparently without ever tiring, every possible aspect of the analysis of elastic structures. He does not spare his time in attempting to eliminate whatever inaccuracy still remains in his methods of analysis by devising procedures of higher precision.

Nevertheless, he invariably turns away whenever it is pointed out to him that the stage in his analysis that could bear some investigation is that which follows moment computation. This is the stage in which, after having obtained statical characteristics such as, for instance, the bending moment for a unit load (by rigorous application of methods of analysis devised on the bases of the best scientific and engineering thought of the last two centuries), he (*a*) multiplies the moment by the specification load, a value generally arrived

NOTE.—This paper by Alfred M. Freudenthal was published in February, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1947, by I. Oesterblom, and John M. Hayes.

[14] Visiting Prof. of Theoretical and Applied Mechanics, Graduate School, Univ. of Illinois, Urbana, Ill.

[2] "The Safety of Structures," by Alfred M. Freudenthal, *Transactions*, ASCE, Vol. 112, 1947, p. 125.

at by the guessing of one or a number of administrators, and then, in order to obtain the required section, he (*b*) divides the result by the permissible stress. This stress value usually represents a rather elusive compromise between the tendency to improve the utilization of the material and the conflicting tendencies to do this without affecting the existing manufacturing processes of the material —without changing established design practices and without, in general, giving the matter too much thought as a fundamental engineering problem.

The question might be asked whether such a procedure constitutes adequate design. However, it appears that this is a rather inappropriate question to ask and, apparently, not many engineers care enough to reply. Why this is so is hard to understand; but there is undoubtedly an intrinsic fascination in playing with numbers, and considerably less fascination in getting behind the numbers to find out how they were arrived at, what they mean, or if they mean anything at all.

It is gratifying to find such wholehearted endorsement of a proposed line of thought as that expressed by Messrs. Oesterblom and Hayes in their comments. In connection with Mr. Oesterblom's very thoughtful discussion, one particularly interesting aspect of the problem emerges. There appears to be one point at which even the most rational engineering mind becomes reluctant to face the consequences of his own way of thinking—this is, when the mechanical strength or the permissible stress of materials is being discussed. Mr. Oesterblom considers Example 2 of the paper, dealing with strength of butt welds, to be of less significance

> "* * * in that it deals with permissible stresses for which the variability largely has been eliminated by more definite and better controlled processes of manufacture. This might not yet be true for a butt weld; it certainly will be true soon. If it were not so, one may well ask if the probability method is in order. If a load were assumed too light, it would not be serious, but if a weld strength were too low at a critical point the result might be disastrous—and to include such a weak point in the setup for probability will not save a structure when it is cursed by such an element of possible breakdown."

This is an admirably concise expression of the (apparently subconscious) apprehension when considering comparative effects of "overload" and "understrength." The variability of the load, leading to a possible overload, is accepted as an indisputable fact. However, if the variability of strength is mentioned, some cord of subconscious fear of "possible breakdown" is struck. Mr. Oesterblom's is not an isolated reaction, but one shared by almost every member of the profession with whom the writer has had the privilege to discuss his views on structural safety.

In his previous paper[2] the writer tried, apparently without great success, to argue the following points:

a. The safety factor is a factor of correlation of "load" and "resistance" (or "strength").

b. By correlation of these two design characteristics, the safety factor automatically compensates for the effects of both ignorance and uncertainty; even if, for the sake of argument, it were considered possible to eliminate "ignorance"

(both subjective and objective), the objective uncertainty inherent in all—even the most carefully planned and performed—human activities and observations remains ineliminable. This applies to both load and strength.

c. It is, therefore, equally impossible to predict an absolute minimum of strength, even if the highest level of control were imposed on the manufacturing process of the material as it is to predict an absolute maximum load. Thus the only rational specification of either load or strength is to specify a value of relatively high frequency of occurrence, with such a range of extreme fluctuation that the probability of occurrence of values outside this range is negligibly small, considering the expected number of loads during the period of service of the structure.

d. It is the probability method that makes it at all possible to correlate an extremely improbable "overload" with an extremely improbable "understrength," by computing, instead of arbitrarily selecting, a safety factor in such a manner that a coincidence of both extreme conditions becomes practically impossible.

e. In view of the preceding arguments design loads and permissible stresses are closely interrelated; thus, in establishing design specifications they must not be considered except in conjunction, precisely because both load and strength can only be specified in the form of statistical frequency distributions. That in this respect there is no difference in principle between the strength of a butt weld and that of a steel section manufactured and rolled under the usual controlled conditions becomes evident by comparing Fig. 4 of the paper under discussion with Fig. 3 of the writer's previously published paper on safety.[15]

There is no alternative approach which can give the designer some confidence that, by rationally specifying an overload not to be exceeded more frequently than once in 10,000 loading cycles, and a minimum strength value to be attained in at least 9,999 of 10,000 tests, the probability of coincidence of extremes leading to disastrous breakdown has been reduced to less than 1 in 100,000,000. It is a chance which, considering the total number of load cycles in civil engineering structures, is beyond any practical range. The writer is particularly grateful to Mr. Oesterblom for having raised the point and thus provided an opportunity for its renewed consideration.

There is no doubt that, as Mr. Hayes points out, the problem is too complex to be solved by the exclusive application of rational methods. It is in this light that the cited remarks by Hardy Cross, Hon. M. ASCE, are very much to the point. So far, however, it is rather the reverse—that is, rational methods have only been admitted in stress analysis. In devising the procedure of how to apply the results of this analysis to the design of real structures, the reliance is mostly on subjective judgment, "experience," and "common sense." It is to plead for the admission, on at least equal terms with subjective judgment and prejudice, of rational analysis in this stage of design—the stage of drawing up of standard specifications—that the writer has presented his arguments in a manner that may appear to place too little emphasis on the existing and very real limitations of the rational approach in engineering.

[15] "The Safety of Structures," by Alfred M. Freudenthal, *Transactions*, ASCE, Vol. 112, 1947, p. 134, Fig. 3.

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DISCUSSIONS

THEORY OF INELASTIC BENDING WITH REFERENCE TO LIMIT DESIGN

Discussion

BY ALEXANDER HRENNIKOFF

ALEXANDER HRENNIKOFF,[26] Assoc. M. ASCE.—Although disappointed with the extent of the response to his paper, the writer is grateful to those few who did contribute to the discussion. The writer appreciates the favorable comments of Professor Winter and agrees with him on the need for a complete set of functions of *m*, *n*, and *u* for the interpretation of test results. A large part of these tables was omitted from the paper to economize on space.

The theory of inelastic bending as presented by the writer is based on the assumption of linearity of the cross sections of the beam, and on the identity of the stress-strain relation in the cross sections of the beam with that in a simple tension and compression test. In this regard, the writer has followed in the footsteps of such recognized authorities as S. Timoshenko and A. Nadai. It is possible, however, that these assumptions do not always hold—in which case the theory becomes erroneous. One such case, involving a mild steel beam, with normal stresses in the plastic range, was discussed in the paper.

Professor Drucker in his thought provoking discussion questions the validity of the assumed stress-strain relation in still another case. The core of his discussion, as interpreted by the writer, is as follows: If in a mild steel beam both the normal and the shearing stresses in all parts of the cross section are within the elastic range, but the principal shearing stress reaches or even exceeds the yield value, then a very large shear deformation would tend to take place in the plane of principal shear. This condition would also result in a very large normal deformation in the cross section, for which no allowance has been made in the theory, since the normal stresses still remain within the elastic range. If, however, this large deformation does not materialize (being arrested by continuity with the adjacent material), then some readjustment in the normal stresses and strains in the cross-sectional planes is bound to occur, and the assumed stress-strain relation also becomes incorrect.

NOTE.—This paper by Alexander Hrennikoff was published in March, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1947, by George Winter, and D. C. Drucker; October, 1947, by Michael R. Horne; and February, 1948, by E. P. Popov, and Bernard A. Vallerga.

[26] Prof., Civ. Eng., Dept. of Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

Although this reasoning is undoubtedly plausible, the extent of the resultant irregularity is uncertain and may very well be small. The writer feels that Professor Drucker's criticism, in order to be accepted as valid, should be supported by an appropriate quantitative analysis or by experimental evidence.

The objections of Professor Drucker apply only to beams made of material endowed with the property of yielding (for example, mild steel), and only in the range of yielding. For this reason, the writer disagrees with Professor Drucker concerning the validity of Section 6 of the theory presented. This section, dealing with the general theory of shearing stress for any material of the beam, is perfectly valid within the limits of the writer's assumptions.

The absence of any explanation concerning the unusual shape of the shearing stress diagram in Fig. 29(*c*) at the fixed end of an I-beam is regretted. Since the shearing stresses in the cross section of a beam are determined by statics as the difference of the normal stresses in the two adjacent cross sections, the diagram indicates that the normal stresses in flanges increase very fast, and that in the web they decrease toward the fixed support. This condition would suggest that the fixation of the end of the beam has been accomplished by holding the flanges and leaving the web free. If this explanation is correct, the diagram merely reflects the effect of some peculiar end conditions induced artificially in the beam and has no bearing on the theory presented.

The question of possible incorrectness of the theory when applied to the mild steel beams in the plastic region of stress, as a result of disturbances produced by the shear effects, is also discussed at length by Professor Popov, who makes several references to other investigations along similar lines.

In the reference to the work of F. P. Cozzone, Professor Popov points out a striking disagreement between the expressions for the shearing stresses derived by Mr. Cozzone and the author. Fortunately, the cause of this disagreement is not difficult to find.

In the first place, the general equation (Eq. 56) for the shearing stress in the plastic region does not reduce to Eq. 58. As Professor Popov points out, in this region $df_m = 0$ and df_0 is distinct from zero, which makes R infinity, and thus reduces Eq. 56 to the form:

$$t = \frac{V\,Q}{I\,b}\,\frac{\frac{A\,C}{Q} - 1}{k - 1} \qquad (75)$$

This error does not however account for the full extent of the difference considered herein.

All the quantities entering Eq. 58*a* refer to the geometry of the cross section, the only exception being the term *V*—the shearing force. Thus, according to this equation, the only characteristic of the loading affecting the shearing stresses in the cross section of the beam is the shearing force. That this conclusion is false in the plastic region of normal stresses in the cross section is evident from Fig. 18. With the normal stresses as indicated in Fig. 18, and quite apart from any particular value of the shearing force present, the conditions of statics demand that there be no shearing stresses on either horizontal or vertical planes in the outer parts of the cross sections, where the normal stresses in the two sides of the element shown are equal, which means that all the shearing stresses must be concentrated near the center of the sec-

tion. Again by statics these shearing stresses near the center must be the greater, the smaller the region over which they are distributed. Thus, it is not only the shearing force, but also the general state of strain, or, in other words, the bending moment, which determine the state of shearing stress on the vertical and horizontal planes in the plastic region of normal stresses in mild steel I-beams—which is definitely not the conclusion which may be formed from consideration of Eq. 58*a*.

The error of Mr. Cozzone's formula is caused by the crudeness of his assumption of the trapezoidal distribution of normal stresses. The free body diagram, similar to Fig. 18, for an element of beam on the basis of this assumption is shown in Fig. 39. The incorrect manner of distribution of normal stresses on the sides of the element of the beam makes it imperative that there be shearing stresses on the vertical and horizontal planes at all points of the beam, and that these stresses must be independent of any conditions of loading other than the value of the shearing force.

Mr. Cozzone's theory is very crude also in some other respects, and it is totally unsuitable for analysis of statically indeterminate beams stressed beyond the elastic limit.

Although Mr. Cozzone's Eq. 58*a* is thus unquestionably erroneous, it does not follow that the writer's Eq. 25*c* and his other equations for the shearing stresses based on this equation are necessarily correct. They are only as correct as the underlying assumption with regard to the normal stresses, and thus the doubts expressed by Professors Drucker and Popov are not unfounded.

In spite of these objections the author still believes that, although the state of strain of a mild steel beam in the plastic region may depart appreciably from the theory presented, the resultant error in stresses is probably small, and the effect of this irregularity on the angle changes, and through them on the statically unknown quantities and the general state of stress and deformation in the statically indeterminate beams, is probably only minor. It should be possible to check this contention by tests.

Mr. Vallerga in his discussion traces the similarity between the method presented by Professor Timoshenko and the writer's theory, thereby raising by implication the question of the originality of the latter. In this connection the writer freely admits that the conception expressed by the symbol m_1 was borrowed from the work of Professor Timoshenko, and the idea expressed by the symbol m represents only a simple elaboration of the former conception, as indeed is shown by Mr. Vallerga. The writer however is not aware of anything in Professor Timoshenko's work, or in any other source, suggesting the equally important ideas implied in the symbols n, u, and q; the extension of the theory into the field of stress recession; and also the general manner of treatment of several problems analyzed by these conceptions—all this comprising at least 90% of the paper.

Mr. Horne has made a thorough study of the theory; he made some further elaboration of it, and solved some additional interesting examples, throwing more light on the behavior of beams beyond the elastic limit. The author, however, disagrees with him on the usefulness of the inelastic version of the three-moment theorem (Eq. 51*b*). In the first place, Mr. Horne is not justified in deriving this formula for a straight beam and then applying it, without further elaboration, to beam ABC (Fig. 31) made of two parts at right angles

to each other. Furthermore, Eq. 51*b* merely complicates the matter, instead of making it simpler. Thus, Mr. Horne applies Eq. 51*b* three times, obtaining three cumbersome equations (Eqs. 53*e*, 53*f*, and 53*g*), from which by elimination of the variables Eq. 53*h* is derived, which he uses in the analysis. Mr. Horne overlooks the fact that an equation equivalent to Eq. 53*h* can be written directly without any intermediate steps by the general formulas in Fig. 11. This equation would be different from Eq. 53*h* only in that it would be based on the basic equations (Eqs. 11 and 17*a*) for ϕ and δ. In order to reach exactly the same form of the equation as Eq. 53*h* (which by the way is unnecessary), Mr. Horne would have to write first the expressions for ϕ and δ that are equivalent to those assembled in Fig. 11, but that are based on Eqs. 13*b* and 18—rather than on Eqs. 11 and 17*a*.

The writer has been much interested in Mr. Horne's discussion on the theory of limit design and on the effect of ignoring the strain hardening. He is in full agreement with Mr. Horne's general outlook on the subject, and with his conclusion that the load beyond which the deformations begin to increase very fast (which can be looked upon as the failure load) depends only very little on whether the strain hardening is operating or whether the yield stress continues indefinitely on further increase in strain. Thus, in Example 3 the value of the failure load H, allowing for strain hardening, is found to be 58.6, whereas a similar value without the strain hardening is 57.2, indicating a negligible difference.

There is, however, an important aspect of the situation which was emphasized strongly by the writer and which was completely ignored by Mr. Horne, although his data present an admirable commentary on it. Table 9 shows that, when strain hardening is ignored, the local unit strain at point D becomes infinite when the value of load H is only 51.03, which is considerably lower than the failure value, 57.2, previously mentioned. Infinite strain can mean only one thing—actual physical failure of the structure at point D. Thus, if strain hardening is absent, the structure fails long before the equilization of moments at points C and D takes place. That which saves the structure from failure is the strain hardening. Thus, Mr. Horne's conclusion (1) and his several statements—to the effect that the behavior of steel structures up to the point of collapse (he means by collapse, large deflections) may be satisfactorily accounted for without any reference to strain hardening—most certainly is not borne out by the facts. A theory disregarding strain hardening indicates an imminent physical failure at a comparatively low value of the load, and does not explain the most outstanding fact—why this early failure does not actually occur.

Thus, it is only with the knowledge that strain hardening does occur and, in the hope that its extent will be sufficient to prevent an early failure, that it is possible to compute the limiting value of the load and the corresponding deformations without any regard for strain hardening. This hope for beneficial assistance of strain hardening is probably nearly always justified when dealing with mild steel structures, but how true it would be in relation to other materials is questionable. There is strong suspicion that with some aluminium alloys, characterized by a very small and slow rise of stress beyond yielding, the theory of limit design in some cases may prove invalid. In presenting the data to serve as a warning for the too ready and unquestioned acceptance of the theory of limit design, Mr. Horne has rendered a good service to the profession.

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DISCUSSIONS

UNDERGROUND CONDUITS—AN APPRAISAL OF MODERN RESEARCH

Discussion

By E. F. Kelley, and Bailey Tremper

E. F. Kelley,[49] Esq.—This is an excellent summary of the results of the research work on underground conduits that has been carried on over a period of years by the Iowa Engineering Experiment Station at Ames, under the direction of Anson Marston, Past-President and Hon. M. ASCE. The writer desires to discuss only Eq. 15, which is presented as a method for computing the horizontal deflection of a flexible pipe, such as a corrugated metal culvert. By making certain assumptions and substitutions, Eq. 15 may be expressed in different form and used to determine the height of fill which will produce a given deflection of a flexible pipe culvert.

In experiments reported by the author[50] it was found that δ, the settlement ratio, ranged from -0.14 to -0.76 for corrugated metal culverts. Elsewhere,[51] as a result of other experiments, the author has suggested settlement ratios ranging from -0.4 to $+0.8$. Therefore, for the purpose of simplification and illustration it is considered sufficiently accurate to assume the settlement ratio equal to zero.

It is apparent from Fig. 8 that when $\delta = 0$, $\delta\,\rho = 0$ and

$$C_c = \frac{H_c}{b_c} \qquad (29)$$

Also, with b_c expressed in feet and r in inches,

$$b_c = \frac{2\,r}{12} \qquad (30)$$

Note.—This paper by M. G. Spangler was published in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1947, by Wilson V. Binger, and Jacob Feld; and February, 1948, by Zdenek Bažant, Jr., Anson Marston, and George E. Shafer.

[49] Chf., Div. of Physical Research, Public Roads Administration, Washington, D. C.

[50] "The Structural Design of Flexible Pipe Culverts," by M. G. Spangler, *Bulletin No. 153*, Iowa Eng. Experiment Station, Ames, Iowa, 1941, p. 56.

[51] "Analysis of Loads and Supporting Strengths, and Principles of Design for Highway Culverts," by M. G. Spangler, *Proceedings*, Highway Research Board, National Research Council, Vol. 26, 1946, p. 198.

Substituting Eqs. 29 and 30 in Eq. 6*a*,

$$W_e = \frac{H_e \gamma r}{6} \quad \text{.........................(31)}$$

Thus, if W_e is expressed in pounds per linear foot of pipe, Eq. 15 may be reduced to the form,

$$H_e = \frac{72 \Delta (E I + 0.061 \epsilon r^4)}{F_d F_k \gamma r^4} \quad \text{..................(32)}$$

Computed values of H_c for corrugated metal pipe having diameters from 18 in. to 60 in., inclusive, are shown in Fig. 28. For the purpose of the computation it was assumed that: $\Delta = 1\%$ of the pipe diameter ($= 0.02 r$);

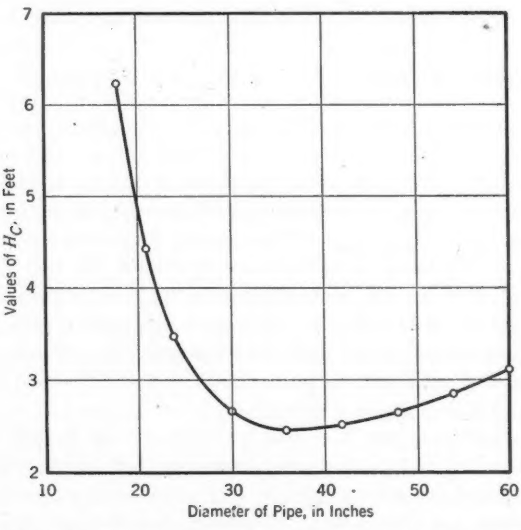

FIG. 28.—RELATION BETWEEN HEIGHT OF FILL, H_e, AND DIAMETER OF 16-GAGE CORRUGATED METAL PIPE, COMPUTED BY EQ. 15 FOR A DEFLECTION OF 1% OF THE PIPE DIAMETER

$E = 30{,}000{,}000$ lb per sq in.; $I = 0.001766$ in.^{4} per in., a value computed for 16-gage corrugated metal; $\epsilon = 20$ lb per sq in. per in.; $F_d = 1.5$; $F_k = 0.10$, corresponding to a bedding angle of about 35°; and $\gamma = 120$ lb per cu ft.

It is quite generally agreed that the deflection of corrugated metal pipes in service should be limited to a maximum of 5% of the pipe diameter. However, as the author has pointed out, Eq. 15 is applicable only within the elastic range of the pipe metal and it is not known that a 5% deflection of a pipe in service is within this range. It is known that, when corrugated metal pipe is subjected to the three-edge bearing test, a deflection of 5% is generally outside the elastic range, whereas a deflection of 1% is well within it. It is for this reason that, for purposes of illustration, the relatively small deflection of 1% of the pipe diameter has been selected.

For the smaller sizes of pipe Fig. 28 shows, as would be expected, that H_e decreases as the diameter of pipe increases. However, the minimum value of

H_c is obtained for a diameter of 36 in.; thereafter the value of H_c increases with increases in the pipe diameter. Such results are unreasonable and cannot be accepted as correct.

There may be one or more reasons for the questionable results shown in Fig. 28. It may be that empirical Eq. 15, although applicable within the range of the author's experiments, may not have a general application. It may be also that the writer's assumption of a constant settlement ratio or a constant value of the modulus of passive resistance, or both, may be incorrect.

These questions, and the further question as to whether a deflection of a pipe in service of the order of 5% is within the elastic range of the pipe metal, lead to the conclusion that on the basis of present knowledge, Eq. 15 cannot be accepted for use in the design of flexible pipe culverts.

BAILEY TREMPER,[52] ESQ.—In the State of Washington there are many pipe culverts installed under embankments of from 50 ft to 75 ft in height and in at least one case of 130 ft. The record of performance of these installations ostensibly should afford an excellent opportunity to check the theories of load and supporting strength as developed by Anson Marston, Past-President and Hon. M. ASCE, and reviewed by the author.

The difficulties in evaluating the theories in the light of such experience become apparent from a study of Fig. 8. The very wide range in loads shown for varying values of projection and settlement ratios requires that these conditions be known with considerable accuracy before proper evaluation can be made.

This discussion will be confined to observations of reinforced concrete pipe culverts in service with no intention of conveying the idea that other types have not been used satisfactorily. In many cases good records are available of the depth and width of trenches in which the culverts were installed. Such trenches, however, usually were dug in steeply sloping ground to one side of the watercourse. The depth varied greatly within short distances and, characteristically, was far from uniform on the two sides of the culvert. A good estimate of projection ratio can be assumed only within wide limits. No information is available on the settlement ratio at any site. Although the three-edge bearing strength of the pipe is known within reasonably close limits, the actual care used in bedding and backfilling, and hence the supporting strength of the pipe in place, can only be assumed.

With no good basis for estimating either the loads on the culverts or their supporting strength in place, definite conclusions as to the validity of the theory are not warranted. Based on careful study of the available data of construction and inspection of the condition of the culverts, it appears, however, that satisfactory service is being obtained under heights of fill greater than that indicated by the theory, and certainly greater than conservative design, based on the theory, would indicate.

The writer, therefore, believes that certain modifications of the theory with respect to loads under high fills and the performance of reinforced concrete pipe culverts are warranted. In so far as is known, the experimental work on

[52] Materials and Research Engr., Dept. of Highways, State of Washington, Olympia, Wash.

which the theory of load is based did not involve fills greater than 20 ft in height. Therefore, data for the curves shown in Fig. 8 for higher fills seem to have been obtained by extrapolation. When the load on the culvert resulting from uniform increments of earth to the partly completed embankment is calculated by the Boussinesq equation, the result for high fills is less than is indicated by the curves in Fig. 8. For example, for embankments having a width at the top of 40 ft and side slopes of 1 on $1\frac{1}{2}$, the loads on the culvert, calculated by the Boussinesq method, are the following percentages of those derived from the curve $\delta\ p = 0$ in Fig. 8:

Fill height, in feet	%
30	95
50	90
70	87

The author has found that the Boussinesq solution is in good agreement with measured forces transmitted by concentrated live loads. Therefore, it should be valid for distributed dead loads.

In estimating the supporting strength of reinforced concrete pipe, the author treats it as a rigid structure, making no allowance for flexibility. Actually after the first crack is formed, the pipe begins to exhibit measurable flexibility. At some point, approximately that of a 0.01-in. crack, vertical and horizontal deformations increase rapidly with increasing load during the test. When installed as culverts, the passive pressure of the adjacent soil must afford considerable resistance to deformation and must therefore contribute appreciably to the supporting strength of the pipe. Flexibility in the pipe tends to reduce the settlement ratio as well, and thus reduces the load reaching the culvert from the superimposed embankment.

Some of the pipe in the culverts, to which reference has been made herein, was found to be cracked to widths of 0.01 in., or somewhat more, when examined a few years after construction. Subsequent inspections have revealed frequent cases of autogenous healing or apparent closing of the cracks. Rarely have cracks, when measured on a second inspection, been found to be of greater width than on the first. These findings lead to a belief that cracks in well-made reinforced concrete pipe culverts, when of the order of 0.01 in., are not harmful. A report published in 1947[53] indicates that serious corrosion of the reinforcement is extremely improbable.

It is suggested, therefore, that reinforced concrete pipe culverts be designed to the full estimated supporting strength of the pipe at a 0.01-in. crack, and that its flexibility beyond this point be considered as an adequate factor of safety. It is not intended to imply that ultimate strength requirements should be eliminated from specifications, as tests to ultimate strength are necessary to establish proper functioning of the reinforcement and capacity to withstand moderate deformation without undue structural damage.

Modern methods of compacting embankments appear to make it unnecessary to install culverts in trenches made for the sole purpose of reducing the

[53] "The Corrosion of Reinforcing Steel in Cracked Concrete," by Bailey Tremper, *Journal*, A.C.I., June, 1947, p. 1137.

projection ratio. Usually the density and strength of the embankment can be made greater than those of the soil in its original position. With proper attention to moisture control and compaction in the areas adjoining the culvert, more favorable conditions can be produced than in a dug trench. This is so because of less width, less settlement at the sides of the culvert, and better opportunity for good bedding. A condition approaching an imperfect ditch results from the usual restriction against heavy equipment passing over the culvert until the fill is a few feet above its top. The zone immediately above the culvert is therefore likely to be in relatively loose condition. The depth of the imperfect ditch, of course, can be increased when found desirable at little additional cost.

It is believed that designs of pipe culverts under assumed incomplete projection ratios are neither economical nor necessary. With reasonable requirements for compaction of embankments, and their adequate enforcement in the vicinity of the culvert, a condition should be obtainable such that the load on the pipe is less than, or does not exceed, that of the weight of the earth directly above it.

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DISCUSSIONS

PROBLEMS AND CONTROL OF DECENTRALIZATION IN URBAN AREAS A SYMPOSIUM

Discussion

BY WILLIAM T. HOOPER

WILLIAM T. HOOPER,[3] Assoc. M. ASCE.—The authors of these two papers take the same view that many city governments in the United States are taking—that cities as we know them are indispensable and provide facilities and advantages that can be obtained nowhere else. Is it not possible that the remedy for the ills to which most cities are heirs is to perform a dissection and allow the individual parts to function as parts of the whole? Instead of trying to correct entirely the mistakes of the founding fathers, it should be possible to guide new growth with intelligent foresight so that new errors are at least separated from the old. The cost will be less in the long run.

Energy, time, and money are being devoted to attempts to prevent decentralization of American cities in order to stop their decay. The programs of rejuvenation consist primarily of expenditures of astronomical sums of money for "super highways" leading from the suburban residential areas into the hearts of the business districts. These sunbursts of expensive pavement are expected automatically to clean up slums, solve traffic problems, and provide for the building of great metropolises.

It seems apparent that the problem of rehabilitation is being attacked from the wrong angle because of the prejudices and shortsightedness of the interests fostering the work. The primary consideration is the desire to protect the huge real estate investments in the business sections of the cities at the expense of the entire taxpaying public.

It can be seen from a study of the growth trends of the average American city that the need for ever-increasing concentrations of population is past. These concentrations are dangerous from the standpoint of defense in war, as

NOTE.—This Symposium was published in November, 1947, *Proceedings*.

[3] Asst. Prof. of Civ. Eng., Northwestern Univ., Evanston, Ill.

well as being inefficient for peacetime operation. The location of businesses in close proximity to antiquated public transportation facilities—freight and passenger, cross-country and local—is unnecessary because of rapid developments in both communication and private transportation.

American cities began as convenient trading and transportation centers on a small scale. The businessman or tradesman lived either in the same building in which he worked, or within walking distance of it. As business increased and individual enterprises became more dependent on others, a centralized commercial area began to form and residences were moved to the outskirts of the district. As public transportation improved, it was possible for the more affluent members of society to move some distance out of the district.

In order to serve the domestic needs of those who were not within walking distance of the larger stores, small shopping centers were formed, which in time were incorporated as municipalities as they attracted more dwellers. Lower salaried workers moved into the vacated dwellings owned by the suburban residents, until gradually urban residential areas were forced farther afield by the expanding businesses. Eventually, the property formerly used for living quarters became more valuable for commercial use and was rented to these enterprises.

With the advent of the multistoried buildings, business ceased growing horizontally and moved upward and inward toward the center again, thus vacating buildings which, in turn, were left to decay by absentee owners. Industries began to grow in the vicinity of the commercial and transportation concentrations, and housing for the swarms of low-paid workers was provided by the old, dilapidated buildings vacated now by both the higher-paid tradesmen and the businesses that had followed them. The aging and the spreading of the blight have continued until in some cases few desirable residential areas remain between these blighted areas and the city limits as defined by the suburbs.

Residents of the small communities have refused to be annexed by the larger cities and thereby be forced to pay for the conditions existing therein. Consequently, the cities have lost the higher real estate taxes on the decent residential property and, in order to obtain revenue, have increased taxes on commercial and industrial real estate. The owners of the business property now feel that their activities must be increased in order to meet their higher costs of operation.

While these adjustments were being made, public transportation was built and expanded in much the same manner as the real estate interests had developed their sphere of influence. No coherent plans were made for future development and, above all, no funds were provided for maintenance and improvement. The result is that public transportation has deteriorated and has become so inadequate that increasing numbers of suburbanites are relying upon private transportation, which in turn has made such rapid development that it is changing entirely the aspect of our modern life. The congestion resulting from the many private vehicles driven to and from work each day has caused traffic problems and created serious hazards which must be eliminated.

Unfortunately, the first and most spectacular solution to these problems to present itself is the express highway, pouring thousands of automobiles into an already overcrowded area each morning and spilling them out each evening. The hope is that business people and shoppers will be enticed into the city for their work and recreation, thus maintaining the business interests established there. Appropriations are being made by which state governments are subsidizing the construction of these arterial highway systems at the expense of citizens who have a steadily decreasing interest in, and need for, the facilities provided by the city. More money will have to be spent to provide parking space on land whose price has been enhanced by the preceding outlay for highways.

It seems inevitable that we must accept the decentralization movement and improve upon its results to spread the burden of taxation and the furnishing of governmental services more equitably. Instead of inhibiting growth by attempting to limit zones in which commerce and industry may grow, it would be more reasonable to permit economic expansion into less densely populated and more conveniently serviced semirural areas.

Existing business could be serviced by the renovation and reconstruction of the substandard housing facilities that in many cases are within walking distance of the central commercial and manufacturing districts. Money spent for express highways could better be spent for cheaper cross-town and interurban access highways planned to facilitate intercourse between small business and residential centers being established outside the urban development.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

APPLICATION OF GEOLOGY TO TUNNELING PROBLEMS

BY F. A. NICKELL, R. H. KEAYS, AND JACOB FELD

F. A. NICKELL,[4] ESQ.—The feasibility of tunnels is dependent on numerous factors, and the geological conditions of the area that affect suitability of the rock are of immediate importance. Rarely, however, are special geological features pertaining to an earlier project so similar in detail that methods adopted to overcome them can be applied strictly elsewhere. Unlike engineering, founded on mathematical principles and largely concerned with materials of prescribed physical properties, geology is not an exact science. Its value to engineering projects results from skilful interpretation based on experience in construction problems. Generalization that limits discussion in technical articles leads to oversimplification and the inferences are not always applicable.

Choice of a tunnel in preference to some other structure is usually determined by economic and engineering data in which geological conditions are a consequential, but perhaps not the most significant, factor. Presumably, the plan would be altered only if insurmountable geological difficulties and greatly increased costs can be demonstrated. Nevertheless, geological proof showing superiority of one route over an alternative line, or the occurrence of questionable ground, including active fault zones, would be of major importance.

The foregoing comment leads to the identification of geological effort in engineering projects, a subject discussed in several published articles. The relationship between the engineer and the geologist has grown harmoniously on the basis that the ultimate responsibility for success of the undertaking rests with the engineer, who would want to make decisions on the best information obtainable. The geologist serves in an advisory capacity, responsible for the disclosure and practical interpretation of all pertinent features of bedrock that affect construction.

The significance of many conditions is influenced commonly by nongeological factors. For example, rock suitable for excavation of small tunnels might not stand well if the tunnel were larger. In addition, the seriousness of objec-

NOTE.—This paper by Ernest E. Wahlstrom appeared in October, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1948, by Berlen C. Moneymaker, and Robert S. Mayo.

[4] Cons. Geologist, Pasadena, Calif.; formerly Head Geologist, Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

tionable items beyond effect on construction must be judged from the viewpoints of repair and maintenance as well as of the magnitude of damage in case of interrupted service. This implies that the geologist ought to know and to evaluate related aspects of the problem.

The amount of study needed depends on the geological complexity of the area, as well as on the length and the size of the tunnel. Investigation is customarily done in stages, starting with general regional examination as a proper background in explanation of local conditions. Detailed mapping of formations along the tunnel line almost always proves valuable providing as it does facts on the distribution of rock types, the nature of doubtful features, and the geologic structure. These can then be projected into approximate position at tunnel level.

Special features are explored by pits, drifts, and core drilling, particularly in the cases of portal locations and of conspicuously bad zones. Ordinarily, the cover of rock above grade is too great for an elaborate program of core drilling down to the tunnel level. In fact, mechanical difficulties, slow progress, and diminished core recovery with depth limit the number of holes that will be drilled.

In most cases, surface mapping provides much of the required geological information on which to base plans for construction. Fracture zones, water-bearing horizons, or elevation of bedrock concealed by thick overburden can be located by one or more geophysical methods of investigation, sometimes with very accurate results.

Geological problems revolve around questions of distribution and physical character of rocks, their probable stability during excavation, and the nature of objectionable features. The fields of interest and responsibility for the geologist, engineer, and contractor may overlap. The geologist should not hesitate to give his views to focus attention on construction problems. The value of suggestions, of course, depends on experience and judgment.

Formations free of defect are probably nonexistent. From a practical standpoint, many kinds of materials can be tunneled easily. Nearly all hard, massive rocks, unless disturbed, excavate readily and stand without support. These include the crystalline group (as granite and gneiss); the indurated, fairly thick-bedded sediments (for example, sandstone and limestone); and many of the lavas, although the latter are commonly jointed and may contain bodies of poorly compacted deposits of fragmental origin.

With diminished hardness, and with consequent lower physical strength, formations of a wide variety become less able to stand unsupported. The ease of excavation is offset by problems of stability, by required changes in construction methods, and by need for protective lining or timbering. These rocks belong to or are analogous to the clayey-shaly series. Sweeping generalization is unwarranted. Adequate recognition of physical limitations will permit safe construction, presumably at a higher cost. In such cases, the geologist, besides identifying the rocks, should propose supplemental treatment to insure successful excavation. Timbering, guniting, or spraying with asphalt will protect the fresh surface.

Unconsolidated deposits—including the normal overburden on bedrock in preparing approach cuts to the portals, valley fill, and alluvial side wash; and the scarcely compacted silts, clay, and coarser debris of stream and glacial origin—commonly offer the most difficult problems. The task is simplified by accurate knowledge of the nature of loose materials and selection of suitable tunneling methods. The presence of some interstitial clay in sandy deposits, mentioned by Professor Wahlstrom, adds to stability, but good drainage and close support are usually necessary. It seems reasonable that the better informed the geologist is regarding construction methods, the more dependable will be his opinions. Underriver tunnels are practical through unstable deposits by use of caissons and compressed air; certain loose materials may be consolidated by grouting beyond the tunnel heading; and stage mining is ordinarily successful in clays (and shales), under which only a section of the tunnel face is removed at one time so that support can be replaced immediately.

Physical features that tend to control the pattern of breakage during blasting operations, or that reduce stability of the rock, deserve particular attention. Massive bodies like granite have nearly equivalent strength in every direction. As a result usually of metamorphism with increasing intensity, the rock assumes a foliate structure, passing gradationally from gneiss to schist, to slate, and finally to phyllite, progressively losing uniformity and coherence. Similar conditions arise from the inclusion of abundant platy minerals such as mica or chlorite. Physical properties are varied according to orientation, and the rocks tend to break along the banding or to slacken on exposure to air. The array of additional problems due to the banded character ranges from overbreakage to caving. Such difficulties can generally be limited by controlled excavation and support. Extremely difficult ground may be found in highly micaceous zones, in which the unconfined material will flow if wet.

The laminar structure of shale and, to a lesser extent, the bedding planes of thinly stratified sediments, facilitate parting on exposure. Initially stable rock may cave after prolonged contact with air, so that a protective asphalt coat is helpful. Tunneling methods are successful with adequate support, much in the manner described for unconsolidated materials. The difficulties increase with the diameter of the tunnel.

Joints are fairly universal in all types of rocks, occuring in sets of different orientation and importance. Weathering is facilitated on the surface by admission of water into the cracks, resulting in accentuation of the patterns. At a depth, beyond the immediate effects of atmospheric conditions, joints become tight. Dependent on prominence, joints have a varied influence on tunneling operations generally expressed by slight overbreakage guided by circumstances that permit dislodgment of rock in blocks outlined by intersecting fractures. That section of the paper describing problems related to joints may be needlessly alarming—unless the excavation is chiefly in the zone of weathered rock. It is doubtful that the alinement in any case would be changed merely to cross the joint pattern at a favorable angle. Practically all tunnel problems traceable to jointing can be remedied by timbering the bad sections and trimming loosened rock elsewhere.

Excavation and stability of formations are influenced by the trend and dip of beds in a stratified series cut by the tunnel. It is usually less troublesome to cross the bedding nearly at right angles, whereas the rock is most secure if the dip is steep or vertical. If the strata are gently inclined or horizontal, the tendency for separation of unsupported masses, with consequent caving, is comparable to the experience with fissile or closely jointed rock. Liberal usage of timbering may be required. The difficulties are greater according to the incompetency of the beds and to the size of the tunnel.

Faults are common in many regions as a result of strong movement between crustal masses. Tunnels are subject to extreme difficulty in excavation of badly mashed rock of the fault zone, particularly where the zone consists of plastic material charged with water. If a fault zone is unavoidable in alinement, the tunnel should be located to cross the feature nearly at right angles to minimize the amount of difficult operation.

The hazards involved in tunneling through faults are twofold: The actual difficulties caused by the instability of the crushed rock, and the questions introduced by the prospect of renewed movement. Excavation can be carried on by methods comparable to those used in unconsolidated materials with adequate support, perhaps by steel liners, kept abreast of the heading. In some instances, stage mining yields results. Grouting to consolidate fragmented rock might stabilize the zone sufficiently for normal tunnel operations. The broken area generally carries considerable water, which must be provided for either by grouting or by special drainage features.

In addition to the practical problems, the geologist must consider the seriousness of future disturbance, which may close the tunnel. The time needed for repair in event of rupture could be disastrous for a metropolitan water system, but might be only a calculated risk for a vehicular tunnel, from which traffic can be diverted. These factors enter into the evaluation of the geological significance.

Variation in geological conditions is endless, so particular problems will inevitably escape the discussion. Nevertheless, Professor Wahlstrom outlines many of the common difficulties encountered in excavation for tunnels. A careful geological study prior to final location and design will afford greater assurance of success, and routine inspection of the work during construction allows both a check on surface interpretation and a modification of ideas required by changed conditions underground. A permanent record of geological features disclosed by the excavation should be filed for reference in the event repairs are needed later.

R. H. KEAYS,[5] M. ASCE.—Greater cooperation between tunnel engineers and geologists and greater respect for the opinions of one another on problems that arise in tunnel construction should result from this valuable paper. Several instances may be mentioned where the lack of correct geological information in regard to the character of the rock has been a great disadvantage in tunnel excavation.

[5] Inspecting Engr., Reconstruction Finance Corp., Brooklyn-Battery Tunnel, New York, N. Y.

Reference has been made in the paper to the Moffat tunnel in Colorado. This tunnel has been outstanding in several ways, particularly so, in that the final cost was several times the engineers' preliminary estimate of cost, which was said to be based on the opinion of an eminent geologist that the rock throughout was solid. On the basis of this preliminary estimate of cost the Colorado legislature had authorized a bond issue of $6,720,000, which included the cost of electrification of the tunnel.

The writer, on taking office as chief engineer of the Moffat Tunnel Commission in July, 1923, was asked by the commission to make an estimate of the cost of the tunnel construction on the foregoing assumption, concerning the equality of the rock. No borings whatever had been made, nor had any test pits been dug to verify the statement of cost. Basing costs on past experience with many tunnels, the writer made an estimate of $5,250,000 for the tunnel work alone. This estimate was to serve as a so-called "upset" price in a proposed fixed fee form of contract. The contract was awarded in September, 1923. In the meantime, on the writer's recommendation, the west portal of the tunnel had been moved south a short distance to the only rock outcrop in the vicinity.

A geological report[6] on the tunnel, after its completion in 1928, designated the formation as that of the pre-Cambrian age. The rock itself has been defined as granite, injection gneiss, and metamorphic granite with some pegmatite, quartz monzonite, and schist for two miles east of the west portal. A badly faulted area of the same kind of rock, approximately one half mile in extent, followed. Gray metamorphic granite with quartz monzonite and injection gneiss seams was encountered the rest of the way—about 3½ miles.[7] The rock proved to be excellent on the eastern side of the continental divide, as was to be expected from the inspection of many outcrops. On the western side of the divide, however, where the rock surfaces were buried beneath unknown depths of glacial drift, the quality of the rock could hardly have been worse for tunnel driving purposes, and it was fortunate that at no place did the glacial drift extend downward to the tunnel line. An estimate of the final cost, made after completion of the work, indicated that the work on the eastern half of the tunnel had not cost more than the original estimate,[6] and that all the extra cost had been on the western half of the tunnel, which was heavily timbered, and, on which, heavy steel supports were used for short distances.

The author speaks of squeezing ground and swelling ground. By these terms it is understood that squeezing ground is mainly the result of pressure on plastic materials, and that swelling ground is that due to chemical changes or absorption of water. It is difficult to admit that the pressures that developed on the tunnel support of the Moffat tunnel for the two miles next to the west portal were due to either of these causes. All the rock was badly faulted and broken into blocks. On removal of the support given by the tunnel rock while in place, these blocks tended to slip out of place and deprive further outlying blocks of their support. It was necessary, therefore, to place tunnel

[6] "Completion of Moffat Tunnel of Colorado," by C. A. Betts, *Transactions*, ASCE, Vol. 95, 1931, p. 334.

[7] "Geology of the Moffat Tunnel, Colorado," by T. S. Lovering, *Transactions*, A.I.M.M.E., Vol. 76, 1928, p. 337.

support immediately after blasting and to maintain it in place, so as not to allow the rock to start moving. An easily compressible local cordwood, loblolly pine, together with some rock packing, was used to pack the space between the timber support and the rock.[8] This permitted additional movement, and no doubt was the cause of some movement which would not have occurred had better packing been used. The writer was in Europe and not connected with the work later on when the worst of the bad rock was excavated in the tunnel.

Another situation wherein a geologist undoubtedly would have been of service occurred on the excavation of the Shandaken tunnel, an important feature of the Catskill Aqueduct.[9] This tunnel was excavated in Hudson River bluestone, a shaly sandstone of excellent quality, and a thin-bedded sandy red shale. These formations were bedded approximately horizontally. In this instance trouble was experienced with the thin-bedded shale which tended to cave up to a point forming a Gothic arch; this type of difficulty was described in the paper (under the heading, "The Tunnel Cross Section").

It was estimated that only a nominal stretch in this 18-mile tunnel would require timbering, but the final result was that nearly 50% of the tunnel length was timbered—practically all due to the thin-bedded shale. This is a result certainly not to be expected in a tunnel which is only 10 ft 3 in. wide and 11 ft 6 in. high inside the concrete lining. This shale was really very dangerous. Construction was carried out by hand methods, mostly using the old top heading and bench system. There were about twelve headings being excavated simultaneously, and the timbering where used had to be kept as close to the heading as possible. This prevented heavy blasting, as the timbering would have been shot out.

The permanent timbering was designed as a three-piece or five-piece set of 10-in. by 10-in. material resting on wall plates which were supported by posts. As an alternate design, a three-piece set was to be supported on hitches cut in the rock walls. Soon both schemes were shown to be impractical, as too much unprotected tunnel existed and progress was altogether too slow. It was a day's job to cut properly the two hitches necessary for one set of timber, and the men cutting them were quite unprotected from rock falls.

At this time the contractor developed a scheme whereby the three-piece set was to be used exclusively, but instead of supporting the timbers on hitches, the inclined legs of the set were each to be supported on a horizontal row of three $1\frac{1}{2}$-in. steel pins set in holes drilled in the rock. On top of these three pins a wooden block was set. The inclined legs of the three-piece timber set were cut on their lower ends in such a way that they would be supported largely by being forced against the rock through wedge action. This was important, as the pins themselves were not intended to take bending moment. The design required that the timbers for each set be specially cut due to the varying width of the tunnel. Those who are familiar with recent developments in tunnel support will recognize that this pin support idea is now in general use, but not for exactly the same purpose as it was originally intended.

[8] "Rock Tunneling with Steel Supports," by R. V. Proctor and T. L. White, Commercial Shearing and Stamping Co., Youngstown, Ohio, 1946.

[9] "The Shandaken Tunnel," by Roy W. Gaussman, *Transactions*, ASCE, Vol. 92, 1928, p. 233.

A good example of squeezing ground was an extensive stretch of altered black chlorite schist in the $8\frac{1}{2}$-mile Boyati tunnel of the Athens (Greece) water supply, constructed from 1926 to 1931. Some of the timber support in this small diameter tunnel had to be replaced as many as four times. It was finally replaced by a concrete block lining, reinforced by thoroughly grouted dry rock packing.

JACOB FELD,[10] M. ASCE.—A rock tunnel is one of the simplest engineering structures for the designer and the draftsman, but it often develops into a troublesome job for the field engineers and supervisory staff. As far as the design is concerned, the location is determined by the geometric requirements of the future use of the tunnel (whether for railroad, highway, or water) and sometimes subject to conclusions drawn from geological explorations and analysis. The actual design of the tunnel section is based on assumed empirical or extrapolated experimental values of the physical characteristics of rock, used in formulas derived from mathematical analysis which are based on the assumption of rock uniformity. That such uniformity never exists is admitted —but any assumption of nonuniformity which results in a much more complicated design analysis will probably be just as far from the actual conditions encountered as the assumption of uniformity.

In regard to construction procedure, the matters which must be determined before work is actually started are, as follows:

1. The question whether a full-face section can be drilled and mucked, and whether the progress must be based on a pilot tunnel with either longitudinal or radial enlargement, or on multiple-step benching, must be answered.

2. Having determined the answer to the first question by guesswork or by other means, the drilling support and bracing erection framework must be designed and procured on an estimated amount of use. In addition, it must be flexible enough to permit rapid modification, if the assumed conditions are not found.

3. The general pattern of drilling (and sequence of blasting) is usually the controlling item of ultimate cost in tunneling. Not only all the required excavation must be moved with a minimum of overbreakage, but the pattern must be such that it can be modified from day to day as the feel of the actual rock is obtained.

4. An amount and a type of explosive to insure economical mucking is chosen so that a minimum amount of scaling will be required and still all the rock will fall in the desired position.

If the geological advice carefully described by the author is proposed for use in the design stage, and if such advice can be of use, then, of course, the paper is of great value. However, the information presented can be obtained in greater detail in a textbook by Robert F. Legget,[11] M. ASCE, and in several others of similar, recognized quality. If the author presents his paper as advice to use when tunneling is actually in operation, most men engaged in such work will agree that the information has come too late.

[10] Cons. Engr., New York, N.Y.

[11] "Geology and Engineering," by Robert F. Legget, McGraw-Hill Book Co., Inc., New York, N. Y., 1940.

It is difficult for the writer to discount the impression that such statements as the author's, defining the division of work and authority between geologists and engineers (under the heading, "The Functions of the Geologist"), are parallel to the dicta one encounters in the construction field (such as the rule that only a dues-paying electrician holding a proper union card may turn on a switch). Similar advice and instruction was recently published to cover highway design and construction.[12,13]

Such a division of professional control of closely allied fields of endeavor can only lead to chaos and an end to progress. The writer received formal training in mathematics and the physical, chemical, and geological sciences; yet, the fact that he received a college degree in engineering has never reduced his interest in geology nor has it discouraged his study of that field, when related to the work at hand. In common with many other practicing engineers, the writer resents the attempt of similarly trained men, who happened to have received degrees under the sponsorship of geology or some other academic faculty, to circumscribe, or in any way limit, the field of his endeavors. In parallel fashion, he considers that men similarly trained can be of service in the engineering field, even though they do not have engineering degrees.

The main part of the paper (under the heading, "Physical Characteristics of Consolidated Rock") contains informative data and conclusions. It should, however, be read by the engineer with the reminder that it is not written by an engineer. For instance, the statement, "Certain rocks are massive or isotropic," indicates that the two descriptions are synonymous—a policy which is not in accordance with recognized engineering terminology. Certainly, anyone who has worked in seamy granite which was massive did not find any isotropic characteristics.

Fig. 6 summarizes the recommendations of tunnel cross sections in caving rock. In Fig. 6(*a*), apparently a uniform caving material is pictured; any shape can be cut through if properly timbered and no shape is safe if it is not timbered. If the indication is intended to represent a uniform rock with no cleavage or seams, then any shape can be cut without timbering. In Fig. 6(*b*), the recommended shape will result after the blasting and the scaling operations are complete, no matter what pattern of drilling is followed; such a condition is immediately taken advantage of in the next round of drilling.

Finally, every tunnel engineer should carefully read and heed the concluding sentence of the paper:

> "In squeezing or swelling ground a circular cross section with closely spaced circular supporting ribs appears to overcome the difficulty to best advantage."

If the reader has been through a similar operation, he should take his own experience preferably as a guide and install any shape of bracing as fast as possible. For, in such ground, it is not feasible to excavate a full section, circular or otherwise, and to expect to place the recommended closely spaced circular supporting ribs.

[12] "Geology and Highway Engineering," by M. T. Huntting, *Transactions*, ASCE, Vol. 110, 1945, p. 271.

[13] "Engineering Geology and Highway Location and Design," by L. W. Currier, Daniel Linehan, and George H. Delano, presented at a meeting of the Highway Research Board, National Research Council, Washington, D. C., December, 1947.

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DISCUSSIONS

SUBSIDENCE OF THE TERMINAL ISLAND-LONG BEACH AREA, CALIFORNIA

Discussion

BY JAMES GILLULY AND U. S. GRANT

JAMES GILLULY,[4] ESQ., AND U. S. GRANT,[5] ESQ.—It appears that this paper contains some fundamental assumptions that are incorrect and that are believed to have led to an unacceptable conclusion. The authors (under the heading, "Factors Contributing to the Subsidence: Pumping from the Wilmington Oil Field") state:

> "As with pumping water from a confined aquifer, reduction in the liquid pressure in the oil sands produces a compensating increase in the intergranular or effective pressures in the soil structure within the stratum affected. Compression of the oil zones results. The amount of compression is influenced by the amount of decrease in liquid pressure and by compressibility of the materials within and bordering the affected zone, at top and bottom."

The writers agree with this statement. However, they wish to point out that it is entirely unproved and highly improbable that the decrease in liquid pressure has been significant except within the oil sands. It is, therefore, gratuitous to assume that it has affected appreciably the interbedded shales.

The authors essentially attribute all the subsidence to compaction of the shales interbedded in the oil sands, and bounding them above and below. They were apparently influenced by an assumption that the sands were relatively incompressible and "probably have properties close to those of concrete." Good concrete, with a low voids ratio and permeability, can hardly be similar to porous, more or less incoherent, highly permeable oil sand, such as occurs in the Wilmington oil field, Long Beach, Calif. If these two materials were similar, there would be little oil and no appreciable flow of oil in the highly productive oil wells. Inasmuch as the authors state that the amount of compression is influenced by the "compressibility of the materials within and

NOTE.—This paper by Frederic R. Harris and Eugene H. Harlow was published in October, 1947, *Proceedings*.

[4] Prof. of Geology, Univ. of California at Los Angeles, Los Angeles, Calif.

[5] Prof. of Geology, Univ. of California at Los Angeles, Los Angeles, Calif.

bordering the affected zones," it is surprising that they failed to test the compaction of the sandstones, because it is only within the sands that certainty is assured of the decline in fluid pressure on which the mechanism appealed to by them depends. They omitted any tests of the sands because they believed the shales to be much more compressible, which, of course, they are, and secondly, because it was easier to obtain undisturbed shale cores than sand cores. Both these reasons are believed irrelevant.

Tests of the sandstone cores by the writers showed them to be significantly compressible, both elastically and by plastic flow—the plastic flow accomplished presumably by movement of cement from points of high pressure to stress shadows between the grains. At any rate, whatever the mechanism of the flow, these tests indicate that the sandstones are sufficiently compressible to account for the entire subsidence without appealing to the very dubious addition of shale compression. Computations for the compression of the sandstones alone yielded a theoretical subsidence about twice the actual—a discrepancy readily explicable, as the pressure drop was necessarily measured at the well points and therefore must be far greater than the average decline through the inter-well areas of the oil sand. The authors' conclusion that the subsidence was due entirely to shale compaction is somewhat surprising in view of the fact that the writers' data and conclusions were available to them in an unpublished report to the Long Beach Harbor Department in 1945.

The correction factor of 50%, derived from the use of the writers' sandstone compaction data, is to be compared with the correction factor of 8% that the authors must use in order to reconcile the theoretical with the actual subsidence in accordance with their theory. In other words, the theoretical subsidence by the author's theory is about twelve times the actual. They apply a pressure decline measured in the relatively permeable oil sands to a great thickness of shales that are known from geological evidence to have a very low permeability, and within which it is very doubtful that any appreciable reduction in fluid pressure has occurred.

It is obvious that the fluid pressure within the shales can decline only by loss of fluid from them. Drilling and electric logs of the wells show that, at Wilmington, as in most other sandstone reservoirs, the oil is confined to the sandstone beds and that the shale beds associated with them are saturated with water. That the shale beds have very low permeability, at least in the direction transverse to the bedding planes, is proved by the fact that this impermeability is responsible for the trapping of the oil through geological time. Furthermore, several water sands have been sealed off from the oil sands by these same, nearly impermeable, "aquicludes." With trivial exceptions, the casings in the Wilmingtion oil field have been perforated only in the oil sands. Accordingly, the pressure decline owing to oil production has primarily occurred within the sand beds, not within the shales. Certainly loss of pressure within the sands has set up a pressure gradient within the shales which must be losing water (and hence compacting by the mechanism advocated by Messrs. Harris and Harlow). It is highly probable, however, that this is a very slow process and that it has not contributed significantly to the subsidence.

The requirement of fluid continuity demands that only as the shales lose

water may the pressure within them decline. Their extremely low permeability, transverse to stratification, indicated by the geological evidence, makes it seem improbable that the water loss could have been significant within the brief period of oil extraction—ten years. The argument need not rest, however, on these grounds; yet, this assumption would be much more likely than that made by Messrs. Harris and Harlow.

The water loss from the shales, upon which their compaction must depend, can be estimated in a way most favorable to the authors' hypothesis, by assuming that all the water produced with the oil came from the shales associated with the oil sands. This assumption is known to be incorrect, because much of the small watercut at Wilmington is produced from sand zones. If it originated in the shales, and not in the edge water or interbedded water sands, such as are revealed by electric logs, it constitutes the most favorable assumptions for their hypothesis. The data on water production are highly significant, for they show that there is no relation whatever between the amount of water produced from a given block of the oil field and the subsidence of the surface over that block.

Water production from blocks III, IV, and V of the Wilmington oil field, from its discovery to January 1, 1945, is shown in Table 1, in which the data

TABLE 1.—Water Production and Subsidence in the Wilmington Oil Field, California

Block No.	Water Production		Volume of subsidence (cu ft)	Ratio of water volume to subsidence volume
	Barrels	Cubic feet		
(1)	(2)	(3)	(4)	(5)
III	202,642	1,134,800	114,670,000	0.0098
IV	6,305,140	35,308,000	101,923,000	0.346
V	2,675,445	14,982,500	103,322,000	0.145

in Col. 4 were supplied by C. L. Vickers, engineer, Long Beach Harbor Department. Data in Cols. 2 and 3 are from unpublished records in the office of the Long Beach Oil Development Company. It seems significant that the subsidence shows no relation to water production, for shale compaction is absolutely dependent on loss of fluid from the shale and this fluid must be water, not oil. These data, together with the geological evidence of very low permeability of the shales, seem to compel the conclusion that the contribution of shale compaction to the subsidence has been insignificant. This is, of course, directly opposed to the authors' hypothesis.

It is obvious that with the decline in fluid pressure and the thicknesses of oil sands, both of which are maximal in the central parts of the oil field, their product, multiplied by any arbitrary constant called "compressibility," is necessarily a maximum in the center of the oil field. Thus, the fact that the authors found a theoretical subsidence roughly proportional to the actual is not surprising, even though they applied the mechanical parameters of a substance that probably did not undergo the postulated decline in fluid pressure, and failed to consider the mechanical properties of the sands within which the fluid pressure is known to have declined.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ESTIMATING DATA FOR RESERVOIR GATES

Discussion

BY GEORGE R. LATHAM, AND D. A. BUZZELL

GEORGE R. LATHAM,[10] Assoc. M. ASCE.—Presented in convenient form, these detailed data based on actual water control gates and hoists should prove of considerable value in estimating costs and preparing preliminary designs. The data have been wisely divided into weights and unit costs to permit the engineer to make proper allowances for the various factors affecting a specific project. The materials included are clearly noted and, as the author states, published data are usually too vague in the material included. In a typical Tainter gate design, the weight of the embedded steel and the anchorage commonly varies from one fourth to one half of the weight of the Tainter gate itself, which indicates the relative importance of the embedded material and of its inclusion in the estimate.

The weights of Tainter gates for private utility projects with which the writer has been associated have been plotted and checked with Fig. 4, and are in close accord with the curve for nonsubmergible gates. However, the weights and costs of the hoists are slightly higher than the values shown in Fig. 5; these would vary somewhat, as the actual hoist construction is dependent upon the requirements of the governing specifications. The data for the Tainter gates and hoists are given separately, thus enabling the designer to determine the relative saving by using a traveling hoist in place of fixed hoists when there are a large number of gates.

For the designer to make a comparison and determine the most desirable gate for a specific project, all suitable types of gates should be considered. This paper covers the usual types of gates with the exception of the common wheeled lift gate with either plain or roller bearings. This popular gate has been proved satisfactory, being used both for crest gates and for submerged intake gates; a pertinent article has been published by G. R. Rich and Ross M. Riegel,[11] Members, ASCE, as well as a discussion by P. E. Gisiger,[12] M. ASCE, on the

NOTE.—This paper by Frank L. Boissonnault appeared in September, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1948, by Theodore B. Rights.

[10] Structural Engr., Ebasco Services, Inc., New York, N. Y.

[11] "Waterways and Gates for Hydro-Electric Plants," by George R. Rich and Ross M. Riegel, *Civil Engineering*, February, 1941, p. 101.

[12] "Economic Design of Waterways and Gates," by P. E. Gisiger, *ibid.*, May, 1941, p. 308.

same topic. Its cost compares favorably with other types, since it is a relatively simple fabricated structural steel gate, and good competition is usually obtained on bids. Published data on costs and weights of lift gates and hoists are also scarce and vague concerning the material included. Information on lift gates in a form similar to that provided in this paper would be of value, and would provide excellent material for a subsequent paper.

TABLE 7.—Cost of Materials per Pound for Tainter Gates and Hoists[a]

Project	Gates				Hoists	
	Size (ft)	Year installed	Cost (cents)	Cost increase (%)	Cost (cents)	Cost increase (%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A	30 × 15	1937 1946	11.2 14.1	 26	18.2 41.0	 125
B	34 × 24	1936 1946	7.2 11.0	 53	22.6 57.0	 152
C	39 × 30	1930 1948	5.65 17.8	 215	....	....

[a] Delivered at job site.

In a period when material and labor costs are skyrocketing the estimator is in a very difficult position to prepare a reasonably accurate cost estimate. The writer has had occasion to install duplicate gate and hoist equipment on several projects and it is believed that the costs given in Table 7 may be of interest. To these prices must be added the erection costs, which indicate that the author's closing comment, "Unit costs in 1946 seem to be about double the 1939 costs" does not err, conservatively speaking.

In closing, the writer wishes to commend the author for an excellent paper.

D. A. Buzzell,[13] M. ASCE.—A valuable tool for the estimator's use in computing the probable weight and cost of certain types of gates is provided in this paper. An excellent model is set up for further investigations along the same or related lines.

It is noted that the vertical-lift crest gate is not covered by the author. The vertical-lift crest gate, in widespread use on larger rivers, is usually a fixed-wheel type from 40 ft to 50 ft wide and up to 50 ft high. Such gates usually are operated with one or more traveling gantry cranes; and, for comparison, it is necessary to take account of the costs of both these cranes and the required heavy operating bridge. In other words, a cost study of a dam spillway, for which either Tainter gates with fixed individual hoists or crane-operated vertical-lift gates are considered suitable, necessitates the preparation of two rather complete designs. This problem arises quite frequently in planning spillways and it is obvious that cost and weight curves given in this paper have only limited application to such a comprehensive problem.

[13] Office, Chf. of Engrs, Washington, D. C.

The type of hydraulic slide gate shown in Fig. 1 has been considerably modified in the recent design practice of the Corps of Engineers, and higher cylinder working pressures are in common use. The air inlet area has been stepped up from 5% to 10% of the gate area depending on water velocity; the sill depression has been eliminated, and many changes have been made in the gate leaf details. The extent of conduit length protected by liner plates has also been reduced. The net effect of these changes has been a reduction in the over-all weight and cost. All-welded carbon steel designs are being considered for this type of gate.

The Corps of Engineers has developed a new Tainter or radial crest gate with inclined side frames and a box girder anchorage which has effected a marked saving in weight over the values shown in Fig. 4. A recent instance in which Tainter gates already contracted for and designed according to older practice were redesigned according to the new standards furnished decisive proof that the latter design would result in pronounced economy.

The fact that future design practice will not be limited to the types of gates discussed in this paper is indicated by a recently proposed installation of Tainter gates in the outlet conduits of a large dam. There would be single 24-ft by 24-ft gates under 180 ft of head in each of seven outlet tunnels, 23 ft in diameter. The indicated saving over vertical-lift outlet gates was $300,000 per conduit or $2,100,000 for the project.

It is regretted that the study was not extended to include caterpillar gates, a large variety of which are used in flood control outlets and power conduit intakes.

Corrections for *Transactions*.—In September, 1947, *Proceedings*, on page 1034, Table 2, line 30, Col. 2, change "88.9 to 98.9"; on page 1035, the denominator of the second term in Eq. 3 should be "10^4"; and, on page 1036, in the title of Table 4, change "Fig. 1" to "Fig. 4."

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DISCUSSIONS

CLASSIFICATION AND IDENTIFICATION OF SOILS

Discussion

BY L. F. COOLING, A. W. SKEMPTON, AND R. GLOSSOP, MILTON VARGAS, DONALD M. BURMISTER, AND M. G. SPANGLER

L. F. COOLING,[14] ESQ., A. W. SKEMPTON,[15] ESQ., and R. GLOSSOP,[16] ESQ.—One of the most important, present needs in applied soil mechanics is for a universal acceptance of at least some broad general classification of soils, and in his paper Professor Casagrande has done much to clarify the position in a field where there is admittedly much confusion. The writers met the difficulties involved in this problem a few years ago when they were engaged in preparing drafts for a British code of practice on site investigations. It was soon realized, as stated by the author, that it is not possible to classify all soils into a relatively small number of groups in such a way that the system would adequately meet the requirements of the many divergent problems of applied soil mechanics. However, it was felt that it would be desirable first to adopt a broad classification which would serve as a basis from which, by expansion and amplification within the original framework, the more detailed and specific classification systems required for various practical problems could be built up. For this reason, the approach was to formulate a general basis for the field identification and classification of soils using a system similar to that presented by the author in Section 15. It was considered sufficient for this purpose to take into account two essential characteristics—the size and nature of the particles composing the soil and the strength and structural features of the soil as it exists in the ground. The type of general arrangement suggested is indicated in Table 8.

Six principal types are recognized on the basis of the size and nature of the particles (Col. 1, Table 8), and the simple tests by which these types are dis-

NOTE.—This paper by Arthur Casagrande appeared in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by Ralph E. Fadum; October, 1947, by James H. Stratton, and Donald J. Belcher; November, 1947, by J. A. Haine and J. W. Hilf, and Jacob Feld; and January, 1948, by Kenneth S. Lane, George F. Sowers, René S. Pulido y Morales, Raymond F. Dawson, and D. F. Glynn.

[14] In Chg. of Soil Mechanics Div., Bldg. Research Station, Watford, England.

[15] Reader in Soil Mechanics, Univ. of London, London, England.

[16] Engr., Messrs. John Mowlem and Co., London, England.

tinguished in the field (Col. 2, Table 8) are similar to those described by the author. In this connection the term "dilatancy" corresponds to the property described by the author under "shaking test."

The subdivision of the uniform, well-sorted sands into coarse, medium, and fine is perhaps an elaboration, but it can be readily made and, since the subdivisions can often be associated with important changes in engineering properties, has much to commend it. The division of clays into "lean" and "fat" can be made on the basis of the extent to which the soil exhibits the typical colloidal properties of plasticity, cohesion, and shrinkage. Col. 3, Table 8, gives the more important types of composite soils. These are primarily natural mixtures of two or more of the principal soil types, but mineralogical variations which have an important influence on the soil properties (mica, organic matter, and calcareous material) are also recognized.

Cols. 4 and 5, Table 8, are concerned with the "in-situ" strength and structural characteristics. Since, in most engineering problems, the strength or density of the soil in its natural condition is of controlling importance and since, similarly, the structural features of the deposit (such as laminations and fissures) can be very significant, the writers would stress the importance of their inclusion in any basic classification.

Finally, the color of the soil should be stated since this often has local significance and is helpful for identification. From the terms defined in Table 8, therefore, it is possible to give a fairly accurate picture of a particular soil in a manner that could be generally recognized. The author's suggestion, that with fine-grained soils a note on the effect of remolding should also be included, is a valuable one.

The writers attach particular significance to the strength classification of clays in all problems except those concerned with near-surface soils, and it would be appreciated if the author would give his opinion on this question. The grouping suggested in Table 8 is based largely on the Boston Building Code with an additional class at each end of the scale. These descriptive terms are implicitly connected with shear strength and, within the writers' experience, they are roughly related in the following manner:

Class	Unconfined compression strength (lb per sq in.)
Very soft	<5
Soft	5 to 10
Firm	10 to 20
Stiff	20 to 40
Hard	>40

From this relation it is possible to develop more specialized classifications for such purposes as foundation bearing capacities and earth pressures.

In cases of near-surface soils and soils used for constructional purposes when the "in-situ" properties are likely to be drastically changed, it is agreed that a classification system leaning more heavily on the composition of the soil is required. Consequently, for site investigations in relation to roads and airfields, the Airfield Classification (AC) System, adopted in 1942 by the United States Engineer Department, was recommended in the aforementioned code of practice.

TABLE 8.—General Basis for Field Identification and Classification of Soils

Principal Soil Types; Size and Nature of Particles			Composite types	Strength Characteristics		Structural Characteristics	
Divisions (1)	Types (2)	Field identification (3)	(4)	Term (5)	Field test (6)	Term (7)	Field identification (8)
Coarse grained, noncohesive	Stones	Boulders; larger than 8 in. in diameter. Cobbles; mostly between 8 in. and 3 in.	Boulder gravels	Loose	Can be excavated with spade. 2-in. wooden peg can be easily driven.	Homogeneous	Deposit consisting essentially of one type.
	Gravels	Mostly between 3 in. and No. 7 British Standard sieve.	Hoggin[a] sandy gravels				
	Sands	Particles mostly between Nos. 7 and 200 British Standard sieves. Particles visible to naked eye. No cohesion when dry. { Uniform / Graded { Coarse—between Nos. 7 and 25 British Standard sieves. Medium—between Nos. 25 and 72 British Standard sieves. Fine—between Nos. 72 and 200 British Standard sieves.	Silty sands Micaceous sands Lateritic sands Clayey sands	Compact Slightly cemented	Requires pick for excavation. 2-in. wooden peg hard to drive more than few inches. Visual examination. Pick removes soil in lumps which can be abraded with thumb.	Stratified	Alternating layers of varying types.
Fine grained, cohesive	Silts	Particles mostly passing No. 200 British Standard sieve. Particles mostly invisible or barely visible to the naked eye. Some plasticity and exhibits marked dilatancy. Dries moderately quickly and can be dusted off the fingers. Dry lumps possess cohesion but can be powdered easily in the fingers.	Loams (silt, sand, clay) Organic silts Micaceous silts	Soft Firm	Easily molded in fingers. Can be molded by strong pressure in the fingers.	Homogeneous Stratified	Deposit consisting essentially of one type. Alternating layers of varying types.
	Clays	Smooth touch and plastic, no dilatancy. Sticks to the fingers and dries slowly. Shrinks appreciably on drying, usually showing cracks. Dry lumps can be broken but not powdered. They also disintegrate under water. { Lean clays show these properties to low degree. Fat clays show these properties to high degree.	Sandy clays Silty clays Marls[b] Organic clays Boulder clays Lateritic clays	Very soft Soft Firm Stiff Hard	Exudes between fingers when squeezed in fist. Easily molded in fingers. Can be molded by strong pressure in the fingers. Cannot be molded in fingers Brittle or very tough.	Fissured Intact Homogeneous Stratified Weathered	Breaks into polyhedral fragments along fissure planes. No fissures. Deposits consisting essentially of one type. Alternating layers of varying types. If layers are thin the soil may be described as laminated. Usually exhibits crumb or columnar structure.
Organic	Peats	Fibrous organic material, usually brown or black in color.	Sandy, silty or clayey peats.	Firm Spongy	Fibers compressed together. Very compressible and open structure.	....	....

[a] Hoggin: Sandy gravel with small admixture of clay. [b] Marl: Calcareous clay.

However, it was considered that this classification system could be legitimately regarded as a development and expansion of the basic descriptive soil classification and that it was in the same category as others designed for special engineering purposes, such as foundation bearing capacities, limits of the applicability of various geotechnical processes (5*a*),[16a] and so on.

The author's discussion on the properties of fine-grained soils and their field identification is a valuable contribution to the subject. His method of using the liquid and plastic limit values for soil identification by use of the plasticity chart has been followed with interest in Great Britain, and Fig. 10 summarizes results on typical British soils.

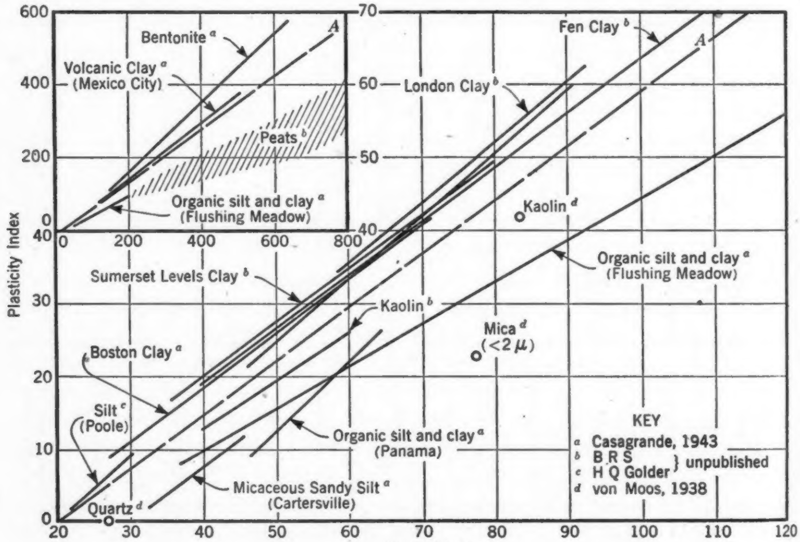

FIG. 10.—RELATION BETWEEN LIQUID LIMIT AND PLASTICITY INDEX

A further relationship for fine-grained soils, which seems to the writers to be significant, is that between the liquid limit and the percentage clay fraction—that is, the proportion of particles of a size less than 2 μ. For samples from a given deposit the liquid limit increases with percentage clay fraction and the relation for some typical British soils is given in Fig. 11. Most clays fall within the zone marked "normal clays." However, the kaolins and boulder clays show quite low liquid limits even for fairly high clay contents. This is probably explained by the "inactive" nature of kaolinite compared with the minerals in most clays and by the fact that the particles of the boulder clay have been produced by mechanical comminution and not by weathering. On the other hand, some of the highly weathered clays and organic muds show a high liquid limit even with a comparatively small percentage clay fraction. This may be indicative of a more than usually "active" clay mineral or the presence

16a Numerals in parentheses, thus: (5*a*), refer to corresponding items in the Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

of unusually fine particles in the clay fraction or, more probably, of a combination of these two effects. In this connection it is interesting to note that the alluvial clay of the River Lea in England has been derived largely from the weathering of the London clay, itself a fairly fat clay, being the product of the weathering and transport of the soil mantle in Eocene times. The indication of the "quality" of the clay fraction which this relationship reveals is considered helpful in assessing the probable characteristics of the soil. As a matter of interest the relationship for seven of the soils given by the author in Table 1, Section 3 are plotted as spot points in Fig. 11. The results agree with the characteristics described with the exception of the diatomaceous earth (No. 5) which, the author states, shows little plasticity although its plasticity index is 42. It would be of interest to know if the diatomaceous earth behaves in an anomalous manner in other respects.

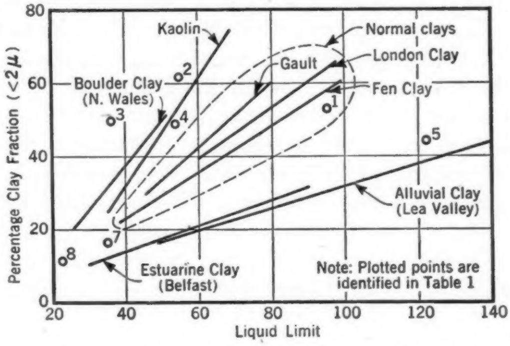

FIG. 11.—RELATION BETWEEN LIQUID LIMIT AND CLAY CONTENT

MILTON VARGAS,[17] JUN. ASCE.—In Section 1 (under the heading, "Development of Systems") Professor Casagrande states that "* * * soil classifications used in civil engineering reflect in their chronological order the development of soil mechanics." This is likewise true of other sciences. Classification systems only reach perfection when the corresponding sciences reach a certain final stage of equilibrium. The existence and success of the Airfield Classification (AC) System shows that this final stage, in the case of soil mechanics, is already in sight.

However, there are some points where explanations are needed. First, in order to be considered perfect, a classification system must be able to divide the things to be classified according to the fundamental characteristics that are significant for the entire class. The AC system begins classifying the soils into two major divisions, according to grain size—the first division referring to coarse-grained soils and the second to fine-grained soils. However, the author was compelled to add a third major division for fibrous organic soils which cannot be classified according to grain-size characteristics. This shows that grain size is a significant characteristic only for a certain class of soils and not generally for all soils. The existence of the third major division without a clear criterion of classification allows the engineer to expect that, in the future, it may be necessary to add new major divisions, more or less arbitrarily, as soils become better known. What then should be the most general

[17] Chf., Soils and Foundations Div., Inst. for Technological Research, Sao Paulo, Brazil.

characteristic of soils for the purpose of classification? That is an open question.

Following the same line of thought, the adopted criterion for separating the major divisions into subdivisions (according to grain-size composition, in the case of coarse-grained soils, and compressibility, or value of liquid limit, in the case of fine-grained composition) appears to be excellent, and the inclusion of intermediate subdivisions among them would do no harm to the system. The same could be said of the groups that divide the major divisions of the coarse-grained soils. However, the groups of the two subdivisions for the fine-grained soils are separated according to two different criteria—grain size and organic matter content. If a new group for the kaolin-type clays is added, as suggested by the author, certainly a third criterion will be introduced—that is, the mineralogical composition.

It would be much simpler, in any future expansion, if the two major divisions of the fine-grained soils were both subdivided into only two groups, that is, the silt group and the clay group, according to grain-size composition. These groups could then be divided into subgroups according to the mineralogical composition and organic matter content, as suggested in Table 9.

TABLE 9.—SUGGESTED SUBDIVISION OF SOIL GROUPS ACCORDING TO MINERALOGICAL COMPOSITION.

1		2		3
Major divisions		Soils groups and subgroups and typical names		Group symbols
Fine-grained soils containing little or no coarse-grained material	Low to medium compressibility	Silts and very fine sands, silty or clayey fine sands, silty clays, slight to low plasticity	Inorganic (except kaolin type)	ML
			Organic	OL
	Liquid limit < 50	Clays of low to medium plasticity, sandy clays, silty clays, lean clays	Inorganic (except kaolin type)	CL
			Kaolin type	KL
	High compressibility	Fine sandy and silty soils, elastic silts	Micaceous Diatomaceous Others	MH
			Organic	OH
	Liquid limit > 50	Clays of medium to high plasticity, fat clays	Inorganic (except kaolin type)	CH
			Kaolin type	KH

The systematic framework of the AC system would thus be improved with practically no change. Thus, the way is open for future additions of new groups without the possibility of confusion.

Another requirement for a perfect classification system is that it be suitable for general use under different conditions. A good soil classification must be as useful for the design of airfields as for foundations or tunnels. The author appears to limit his classification system in referring to it as an Airfield Classification System. Its applicability seems to be very much wider.

In his paper the author does not explain whether the group symbols he proposes are intended to be simple initials of the group names, in order that the users may easily remember them, or whether his intentions were to propose symbols of world-wide significance for soils, in the way that chemical symbols represent the chemical elements. In the first case it would be logical that the group symbols should be changed in each language; on the other hand, in order to maintain the universal significance of the symbols they should not be translated. This would be really advantageous for the proper understanding of engineers all over the world—at least as far as soil properties are concerned. The fact that the group symbols are initials of English words would be no impediment when one realizes that the English language is as widely recognized today as Latin was in the days when botanic nomenclature or chemical symbols were established.

DONALD M. BURMISTER,[18] Assoc. M. ASCE.—A worthwhile contribution to the engineering profession has been made in this critical review of the soil classification systems now in use and in the presentation of the Airfield Classification (AC) System, which served such a useful purpose in the work of the Corps of Engineers, United States Army, during World War II. It is significant that the AC system has been developed on the basis of the important behavior characteristics of soils, and therefore has a broader application than to airfield construction alone.

The greatest practical use of the AC system in the postwar period is in the preliminary exploration, design, and construction stages of projects involving earthwork and foundation problems, where the information obtained is put to immediate and practical use. Those engaged in the soil engineering work on a given project become intimately familiar with the class designations and symbols of the different classes of soils encountered, and they are able to infer accurately the general qualities and behavior characteristics. The group symbols become a convenient tool and expedite the work wherever a large number of samples must be examined and identified. The author has properly warned against the abuse of classification symbols wherein only the group symbol is given without more specific information. Use of symbols alone short-circuits the process and assumes the triple role of identification, description, and classification. After a lapse of some time following the completion of a project such a classification of the soils encountered on a project inevitably becomes more and more vague and less significant as to the actual character and behavior of the soils, because symbols or class designations can never convey a sufficiently precise meaning.

For all permanent records of an organization engaged in soil engineering work, as well as for presentation of information in papers and articles in the technical journals, the more precise and significant descriptive classification, such as that discussed by the author, should be used so as to convey a definite meaning to those who have not had the benefit of examining and working with the soils firsthand. The greatest field of progress in soil engineering lies in the accumulation of detailed, accurate information on the physical character

[18] Associate Prof. of Civ. Eng., Columbia Univ., New York, N. Y.

and properties of soils and on their behavior under the varied conditions encountered in practice. Thus, a body of authoritative knowledge on soil behavior, which would be of great practical value and use, can be built up, not only in a given organization but also for the profession as a whole. Therefore, in the interest of greatest progress in soil engineering, and of maximum usefulness of data on soil behavior, greater emphasis should be placed on the development and continued use of thoroughness in examining, identifying, and describing soils. These considerations far outweigh any consideration of convenience.

In order to satisfy these essential requirements and needs in the postwar period the base of the AC system should be broadened to make the system more fundamental, and at the same time more precise and significant for all types of foundation and earthwork problems. There are two basic steps in such a process.

The first step is to come to some agreement on the definitions of soil components, which may find general acceptance and which may form a common basis for work in soil engineering. Recommendations have been presented by the Foundations and Soil Mechanics Committee, Civil Engineering Division, American Society for Engineering Education (A.S.E.E.), for the consideration of those interested in soil engineering work (24). These definitions are intended to define terms to be used in identifying and describing soils, and are not a textural classification. The following is quoted from the report of this committee:

> "The definitions of the principal soil components: GRAVEL, SAND, SILT, and CLAY should be based on significant criteria. From an engineering point of view the primary difference between sand and gravel is the size of the constituent grains, which can be recognized visually. The primary differences between sand and silt are that the constituent particles of silt cannot be readily distinguished by the unaided eye and that silt exhibits considerable capillarity. The significant and distinctive difference between silt and clay is that clay exhibits plastic properties and silt does not. In the case of fine-grained soils containing clay the influence of grain size is dominated by the influence of mineralogical and chemical composition. Thus, gravel and sand should be defined on the basis of grain size; sand and silt should be defined on the basis of grain sizes and of capillarity; while silt and clay should be defined on the basis of plasticity. These universally comprehended terms form an adequate and satisfactory basis for the definitions of soil components."

In order to have a workable system of definitions for practical use, it was considered desirable by this committee to define the size limits of the principal soil components and of the coarse, medium, and fine fractions directly in terms of sieve sizes, which convey a definite idea of particle size. The term "CLAY-SOIL" is used instead of "CLAY" in order to be more precise in terminology, because the silt admixture cannot be separated out to determine the proportion of the true clay fraction. The A.S.E.E. recommended definitions are given in Table 10. [Capitalized terms, as "CLAY-SOIL," and soil names are given in quotation marks throughout this discussion to indicate that they are proposed special terminology for describing and naming soils.—Ed.]

The second step is to establish a common language for description of soils so that the soils examined and identified can be properly named and described in simple engineering and technical terms. The author points out that at present there is no uniformity in the terms used to describe soils. The descriptive terms, however, should be sufficiently precise and accurate, and at the same time sufficiently flexible in nature, so that the entire range of natural soils can be readily described. The soil name should have a clear, well-defined meaning to enable people who have not had the benefit of examining the soils firsthand to readily understand and to interpret them. These descriptive terms should be suitable, not only for rapid field identification by visual and manual means, but also for the more detailed laboratory identification based on soil tests.

TABLE 10.—AMERICAN SOCIETY FOR ENGINEERING EDUCATION RECOMMENDATIONS FOR SOIL COMPONENTS AND FRACTIONS

Principal components (1)	Description (2)	Sieve limit (3)	SIEVE SIZES FOR SUBCOMPONENTS: Coarse (4)	Medium (5)	Fine (6)
BOULDERS and ROCK[a]	Retained on 3-in. sieve	Lower	....	....	3 in.
GRAVEL and STONE[a]	Passes 3-in. sieve; retained on No. 10 sieve	Upper Lower	3 in. 1 in.	1 in. $\frac{3}{8}$ in.	$\frac{3}{8}$ in. No. 10
SAND	Passes No. 10 sieve; retained on No. 200 sieve	Upper Lower	No. 10 No. 30	No. 30 No. 60	No. 60 No. 200
SILT	Passes No. 200 sieve; nonplastic, little or no strength when air-dried	Upper	No. 200	....	....
CLAY-SOIL	Passes No. 200 sieve; exhibits plastic properties and clay qualities within a certain range of moisture content; considerable strength when air-dried	Upper	No. 200	....	....

[a] BOULDERS and GRAVEL refer to waterworn material; ROCK and STONE refer to angular fragments.

The descriptive terms shown in Table 11, which have been used and found to be satisfactory by the writer, are suggested. Such terms have been used to a considerable extent by field engineers and boring foremen, but without having the precise, well-defined meanings given in Table 11. Although natural soils are composite materials composed of various proportions of the principal soil components in almost infinite variety, recognizable proportions can be defined for practical use. Descriptions of typical soils are given at the end of this discussion to illustrate the use of this terminology in naming soils.

The proportions given in Table 11 cover recognizable ranges, sufficiently obvious for practical purposes. In borderline cases between two proportions either designation is acceptable. The descriptive terms for these recognizable ranges of proportions are used instead of percentages directly because they form a more suitable soil name, which is sufficiently precise for all practical purposes, considering the variation to be expected from sample to sample in

the same deposit. The proportions of the gravel and sand components can be readily determined visually; the proportions of the sand and silt components can be determined by feel and by simple manual tests. In the laboratory or in the field office a standard set of bottle samples should be made up for reference. Thus, a person who is not continuously working with soils and identifying them can refer to these standards and check himself, until he has again acquired the sense of identification and of proportions, which really involves seeing and comparing volumes, at the same time giving due consideration to weight and size of grains, and other characteristics.

TABLE 11.—DESCRIPTIVE TERMS FOR COHESIONLESS SOILS TO BE USED IN FORMING THE SOIL NAME

Soil component (1)	As written in the soil name (2)	Descriptive or qualifying terms as written (3)	Range of proportions (4)
Principal	GRAVEL, SAND, SILT	....	50% or more[a]
		and	35% to 50%[a]
Others	Gravel, Sand, Silt	some	20% to 35%[a]
		little	10% to 20%[a]
		trace	1% to 10%[a]
Subcomponents		coarse to fine	all sizes
		coarse to medium	<10% fine
		medium to fine	<10% coarse
		coarse	<10% medium and fine
		medium	<10% coarse and fine
		fine	<10% coarse and medium

Additional descriptive terms:
(1) Color, grain shape, etc.
(2) Degree of compactness, degree of plasticity.
(3) Inorganic constituents (mica, shells, and foreign matter).
(4) Organic matter (roots, humus, peat, and muck).
(5) Geological origin (alluvial, glacial, wind, beach, swamp, etc., also horizon).

[a] Finer than, or coarser than, the principal soil component.

Such a set of standard bottle samples should be composed of at least fourteen combinations covering the limits of the proportions corresponding to the descriptive terms in Table 11: (*a*) Seven of dry gravel-sand mixtures in the percentage proportions 90–10, 80–20, 65–35, 50–50, 35–65, 20–80, and 10–90; (*b*) three of dry sand of coarse, medium, and fine sizes, respectively; and (*c*) four of moist sand-silt mixtures in the percentage proportions 90–10, 80–20, 65–35, and 50–50. All samples should be at least one pint in volume, and preferably larger.

In examining and identifying soils one should start with the coarsest component and work by stages to the finest component. The proportion of the gravel component and its character as to coarse, medium, and fine fractions should be determined on a fairly large sample. The sand component and finer fractions are then examined to determine the proportion and character of the sand component and of the silt or clay-soil component. Visual means are satisfactory for sands containing not more than a trace of silt. The fine-grained soils or fractions of a soil are more difficult to identify; it is also more important to identify them accurately. Simple manual tests, such as the shaking test

for silty soils and the plasticity test (thread rolling test) for clay-soils, can be used satisfactorily for proper identification by adjusting the moisture content to some particular condition. Plasticity and certain clay qualities are the most distinctive and characteristic properties of clay-soils. Therefore, they form a satisfactory and significant basis for identifying and naming these soils. Table 12 lists suggested descriptive terms based on the degree of

TABLE 12.—DESCRIPTIVE NAMES OF CLAY-SOILS BASED ON THE DEGREE OF PLASTICITY[a]

Degree of plasticity (1)	Plastic index (2)	Descriptive name as written (3)	Qualities (4)
Nonplastic	0–1	SILT	Friable
Slight plasticity	1–5	trace Clay	Desirable
Low plasticity	5–10	little Clay	Cohesiveness
Medium plasticity	10–20	CLAY and SILT	Increasingly objectionable plastic displacements and compressibility
High plasticity	20–35	Silty CLAY	
Very high plasticity	>35	CLAY	

[a] Over-all plasticity of sand-clay-soil fraction.

plasticity of the sand-clay-soil fraction to be used in forming the soil name. These terms have also been used and found to be satisfactory by the writer for a number of years.

For a field and laboratory identification the over-all plasticity of the sand-silt-clay proportion of the soil as determined by the simple plasticity test is the most significant property, because it has a more direct relation to the behavior characteristics of the soil, and because a separation is not conveniently possible. The gravel sizes can be picked out by hand for the plasticity test. The range of the degree of over-all plasticity expressed in terms of the plasticity index for each clay-soil type in Table 12 is broadened and adjusted to be more suitable and significant for identifying the soils. The proportion of sand can be determined by the feel of the soil.

The author has given a few examples of descriptive soil classification to illustrate how the AC system can be expanded. The descriptions do not, however, convey a sufficiently definite meaning to a person working in another field of soil engineering, who has not had the benefit of examining the soil firsthand, but who wishes to use the information on the behavior of such soils for the practical purposes of sizing up and judging his own particular problem and for reaching a satisfactory solution. Examples 1, 2, and 3, hereinafter, illustrate some advantages and the general usefulness for all purposes of the descriptive names of soils, based on the terminology in Tables 10, 11, and 12. It is intended that the soil name be put together in a certain definite fashion to emphasize the significant characteristics of the soils examined. The capitalized term in the soil name stands out as the principal component of the soil, so that it can be spotted at a glance. The significant detail then follows, expressed by the qualifying terms giving the proportions of the other soil components, usually in order of their importance. As is frequently necessary

one can glance down a boring record, soil profile, or a list of soils from an area, and pick out immediately the types which have certain significant characteristics bearing directly on a given problem. The descriptive names of the soils can be abbreviated somewhat for a large group of similar soils, provided that the upper and lower limits of the group are given in detail.

Example 1.—

(*a*) "Light tan coarse to fine GRAVEL, and coarse to fine SAND, trace Silt." (Maximum size $1\frac{1}{2}$ in.; plus No. 4 sieve, waterworn.)

(*b*) "Light brown coarse to fine GRAVEL, some coarse to fine Sand, trace Silt." (Maximum size $1\frac{1}{2}$ in.; plus No. 4 sieve, waterworn.)

Certain important conclusions can be drawn from the descriptions of these soils based on a general knowledge of the behavior characteristics of different classes of soils. First, the soils will be free draining because there is not more than a trace of silt. Second, the soils ranging from "GRAVEL and SAND" to "GRAVEL, some Sand" (as an abbreviated descriptive classification) represent about the limits of drainable soils that can be considered well-graded. Third, the large gravel content and the large sizes indicate that high supporting capacity can be expected in the compacted condition. The quality of being well-graded is significant only for soils containing a large proportion of gravel, not only because it connotes compactibility to high densities (exceeding 135 lb per cu ft), but also because the close packing and contacts, together with the interlocking and proper bedding of the large particles, provide a high supporting capacity.

Example 2.—

(*a*) "Light brown coarse to fine SAND, trace Silt"; medium compact.

(*b*) "Light brown medium to fine SAND, little Silt"; medium compact.

(*c*) "Light brown medium to fine SAND, some Silt"; loose.

From these descriptions certain significant characteristics are immediately apparent. These soils are nonplastic in character. Soil (*a*) is free draining, has only slight capillarity, and is not susceptible to frost heaving. Soil (*b*) is not as free draining, but can be drained by well points, has some capillarity, and is slightly susceptible to frost heaving with high ground-water levels. Soil (*c*) is more difficult to drain with well points, has rather objectionable capillarity, and has frost heaving characteristics. On the basis of natural compactness, soil (*a*) would be much superior to soil (*c*) in the natural state as to supporting capacity. On the other hand, soils (*b*) and (*c*) would show better compactibility because the silt fills the void spaces and provides a bedding for the sand.

Example 3.—

(*a*) "Brown medium to fine SAND, some Silt, trace Clay"; slight plasticity.

(*b*) "Gray-brown SILT, some medium to fine Sand, little Clay"; low plasticity.

(*c*) "Gray CLAY and SILT, little medium to fine Sand"; medium plasticity.

All these soils are relatively impervious. Soil (*a*) would have fairly good compactibility. The supporting capacity would be only slightly affected by capillary saturation in the compacted state. Soil (*b*) is on the borderline of

objectionable plastic displacement characteristics and softening by capillary saturation in the compacted state, particularly with a high ground-water level. Soil (*c*) would compact to a low density and would definitely have objectionable characteristics. However, this soil might have a fairly good supporting capacity in the natural state, if the natural consistency was stiff or better.

The amount of extra effort and time required to identify and describe soils in the more thorough fashion described herein is small when compared to the greater usefulness and value of the information obtained for all purposes, especially to people who wish to use such information to aid the solution of their own particular problems. Such descriptions and soil names, if generally used in boring records and in soil profiles and soil sections, would give far more significant and valuable information to the designing engineer, and, as a part of the contract drawings, to the contractor and construction engineer, than is obtainable at present from the vague and often wholly inaccurate terms generally used. Such information can be interpreted, as noted in the foregoing examples, on the basis of a person's particular experience, into terms of significant behavior characteristics, which would have a more direct bearing on the solution of the practical problems of foundation design and of the practical problems of foundation construction. If a typical soil near the upper limit and another near the lower limit of a group of soils, which have in common similar characteristics, are tested in detail in the laboratory to determine all the pertinent physical properties and strength characteristics, and the results are reported with the descriptive classifications of the whole group, then the range of behavior characteristics to be expected for the entire group can be fairly reliably known. When the actual behavior and performance of the different groups of soils have been observed in sufficient detail in the field and have been correlated with significant soil characteristics, the information would become an important part of a body of authoritative knowledge, which would be extremely useful and valuable to the engineering profession.

M. G. Spangler,[19] M. ASCE.—The fundamental purpose of a soil classification system is to provide a language by means of which one person's knowledge of the general characteristics of a particular soil can be conveyed to another person or group in a brief and concise manner, without the necessity of entering into lengthy descriptions and detailed analyses. Since soils are so widely heterogeneous in character, it is not to be expected that one kind of classification system can cover all the possible features of a soil which it may be desirable for one person to convey to another—hence, the desirability of several bases for classification. In engineering work, it is frequently appropriate to use at least several of, or all, four classifications for describing soils, such as: (1) The geological classification, which provides a language for stating the geological history and background of a soil; (2) the textural classification which provides names for describing the relative proportions of sand, silt, and clay size particles in a soil; (3) the pedological system of classification which reveals the nature of the soil profile as affected by the climatic and other environmental conditions

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under which the profile developed; and (4) an engineering classification that groups soils on the basis of their characteristics which influence engineering performance.

Great care should be exercised not to read into these various classifications more information than they are intended to convey. Particularly, caution is urged against leaning too heavily upon soil classification as a basis for design, since many of the factors that influence design cannot be incorporated into an engineering soil classification. For example, the susceptibility of a road or runway to frost damage is a question that involves not only the properties and characteristics of the soil itself, but climatic and geophysical factors as well.

With reference to the grain-size scales used in various branches of applied soil science, it would be more convenient for civil engineering usage if the arbitrary division between silt size and sand size were established at 0.074 mm (the size of opening of a standard No. 200 sieve) instead of sizes 0.02 mm, 0.05 mm, or 0.06 mm, shown in the various scales quoted by the author in Fig. 2. This is for the purely practical reason that a No. 200 sieve is about the smallest size that can be used in routine soil testing. Sieves with much smaller openings are manufactured, as is well known, but they are much more delicate and expensive and will not wear under rigorous usage as well as the No. 200 sieve. At present, many engineering organizations distinguish between coarse-grained soils and fine-grained soils on the basis of the quantity of material passing the No. 200 sieve, and it would be convenient if this size also marked the boundary indicated by the terms sand size and silt size.

The author mentions the difficulty involved in the fact that textural classifications based on the percentages of sand, silt, and clay size in a soil do not reflect all the physical properties of interest to the civil engineer. The term "texture," as originally used in the United States (53) was not intended to convey any information concerning a soil beyond the relative proportions of the size groups of the material less than 2 mm in diameter. It was not intended to indicate such properties as plasticity, shrinkage, swell, and others. Rather, those additional properties should be expressed by other kinds of classification or by specific test results. The term "texture" is used in some European countries (German *textur*) to embrace properties of soil other than relative amounts of the various size groups, such as soil structure and the form and relative size of the void spaces, but in the United States, so far as this writer is aware, "texture" has always referred only to grain-size relationships. It is the writer's belief that less confusion will ensue if this meaning of "texture" is preserved and emphasized.

No discussion of triangle textural classification is complete without pointing out that all the information given on an equilateral classification triangle can be shown on a right triangle (54) and that the process of determining the textural class of a soil is much simplified by using the right triangle chart. Since the sum of the percentages of sand, silt, and clay size in the portion of a soil smaller than 2 mm is 100%, the values of only two of these percentages are required to establish a point in an equilateral triangle such as shown in Fig. 3. By virtue of this fact, a right triangle can be constructed, as shown in

Fig. 12, on which the percentages of any two of the three size groups can be scaled along the legs of the triangle. The point of intersection of the percentage lines representing the two groups for which the triangle is constructed can be more quickly spotted on such a diagram because of the orthogonal arrangement of the coordinates.

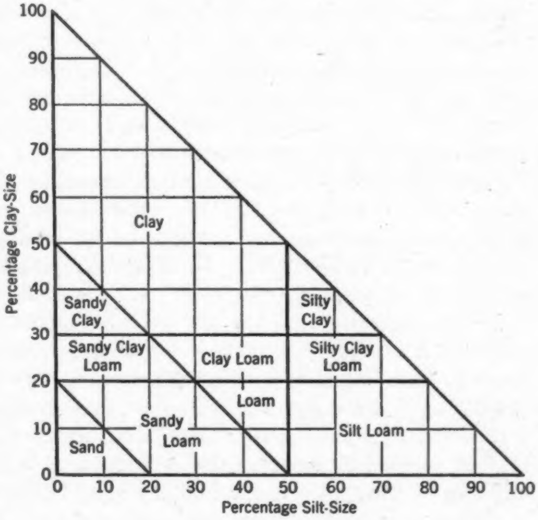

FIG. 12.—RIGHT TRIANGLE TEXTURE CHART

Soils that contain sufficient material larger than 2 mm to be especially noticeable (say, about 8% or 10%) should be qualified by prefixing the term "stony" or "gravelly" to the textural class indicated by the triangle chart, as stated by the author. The principal textural class name of such a soil is determined on the basis of that part of the material smaller than 2 mm. Obviously if a soil contains an appreciable quantity of +2 mm material, the percentage that each of the three smaller sizes bears to the total soil will not be the same as the percentage of these groups in relation to the amount of the sand, silt, and clay only. It is this latter percentage which should be used in the triangle chart classification. The percentages relative to the whole soil can be quickly converted to the percentages of the sand, silt, and clay by multiplying each of the original percentages by the ratio, $\frac{100}{100 - \text{gravel } (\%)}$.

Thus, if the mechanical analysis of a soil shows it to consist of 17% gravel, 26% sand, 36% silt, and 21% clay, the ratio by which the percentages of the three smaller sizes should be increased is 1.2. The new values for these sizes then become 31% sand, 43% silt, and 25% clay. Entering the triangle chart (Fig. 3(*a*)) with these revised figures, the point of intersection falls within the clay loam area and the textural classification of this soil is "gravelly clay loam." If the material larger than 2 mm consists of broken stone fragments instead of rounded gravel particles, the soil would be classed as a "stony clay loam."

If one attempts to use the original percentages of sand, silt, and clay (that is, the percentages referred to the whole soil) three different points of intersection can be obtained on the equilateral triangle chart, depending on which pair of the three sizes is used. In the foregoing example 26% sand and 36% silt gives a point within the area labeled clay; 26% sand and 21% clay gives a point in the silty clay loam area; and 36% silt and 21% clay yields a point in

the clay loam area. The necessity for reducing the percentages to the weight of material smaller than 2 mm as a base is apparent.

This writer has no particular criticism relative to the Airfield Classification (AC) System proposed by the author and tentatively adopted by the United States Engineer Department. It is a good system of classification. He does feel, however, that it is unfortunate for the civil engineering profession as a whole that the Office of the Chief of Engineers deemed it necessary to adopt any entirely new system for classifying soils for engineering purposes. Here is a new "language" added to the existing Babel of tongues which must be learned, and a new source of confusion to be surmounted, if an engineer is to avoid pitfalls in the matter of soil classification, as suggested by the author in the opening paragraphs of his paper. There would be no objection to the necessity for learning this new language if the need for an additional classification system were clear cut and positive. However, careful study of the AC system fails to reveal any substantial superiority over the revised Public Roads (PR) System of soil classification. They are tweedledum and tweedledee in many major respects. Viewed objectively it seems somewhat specious to argue that a soil needs to be classified by a different system just because the wheel loads to which it will be subjected are those of an airplane instead of a truck; and, in view of the fact that the Civil Aeronautics Administration also has its own classification system, one might conclude that the needs of the situation are different if the airplane wheels are those of a military plane rather than those of a civil aircraft.

There is need here for "engineering statesmanship" to bring these three federal agencies closer together in the matter of engineering soil classification. Without doubt, the problems of the three agencies are different and must be solved in different ways, but all have a common interest in engineering properties of soil. A great deal of "compartmentation" and confusion could be eliminated if all three would use the same language when discussing those properties.

The revised PR system of soil classification provides an excellent medium for expressing the general engineering characteristics of soils with sufficient clarity for practically all engineering applications and with a degree of precision commensurate with that which can be expected of any classification system. It has been used by engineers since about 1928, and on an ever widening scale, until nearly all who deal with the soil have a general understanding of the meaning of the symbols A-1, A-4, and others. It is true that local engineering organizations often have had to modify the system to fit their local situations but, by and large, such modifications were readily made within the framework of the basic system, so that the language was essentially preserved.

It is also true that the PR system has undergone a number of revisions since its introduction, which is to be expected in the case of any new tool or device as use demonstrates the need for modification. The author states that the AC system has been revised three times since its inception in 1942. Undoubtedly both systems will undergo further revision as time passes.

The most recent and the most extensive revision of the PR system is that which has been made by a committee of experienced engineers designated by the Highway Research Board, under the chairmanship of Harold Allen, M.

ASCE, of the United States Public Roads Administration. The findings of a highway subcommittee of this committee (51) constitute a major revision in the PR system which brings it up to date in the light of many years of experience. This revision retains the basic framework of the system, with changes in detail here and there which are clearly set forth in the report. In other words, the classification language has not been changed, but the meaning and connotation of some of the terms has been modified as dictated by experience.

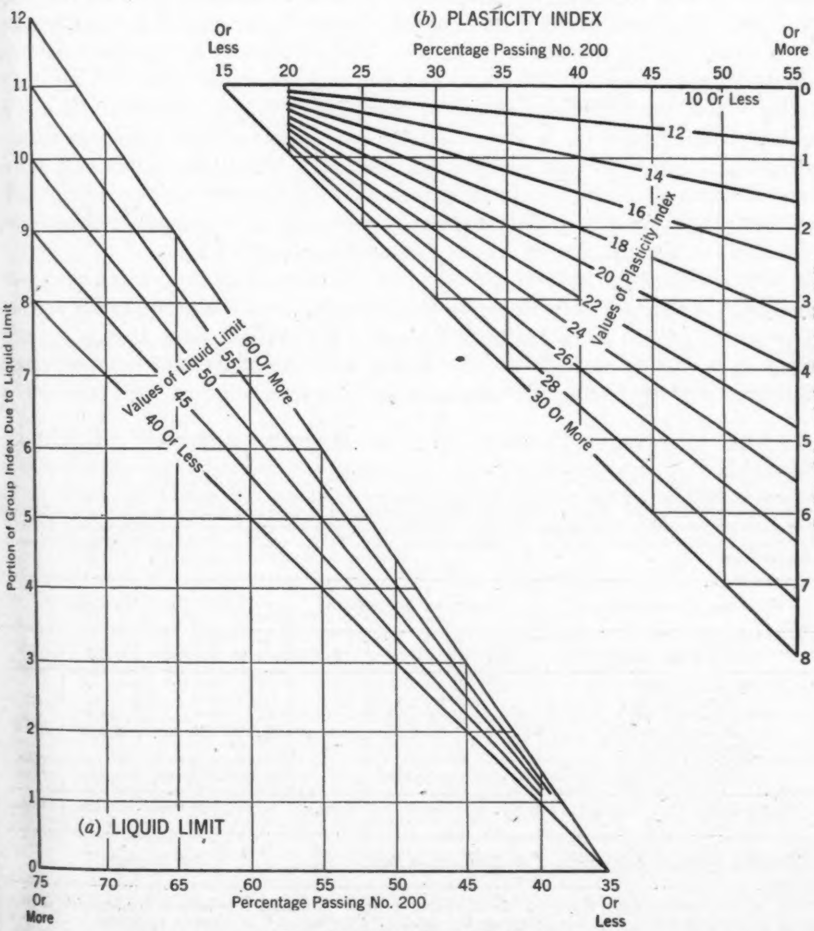

FIG. 13.—GROUP INDEX CHARTS

A major accomplishment by this committee has been the simplification of the procedure by which a soil may be classified. It is now possible to classify a soil, quickly and definitely, into one of twelve major groups and subgroups on the basis of three properties of the material—the mechanical analysis, the liquid limit, and the plasticity index. The committee did not minimize the

value of other test constants such as shrinkage factors and centrifuge moisture equivalent in connection with design procedures, but felt that they were not essential in a simplified classification procedure.

Another major achievement by the committee was the introduction of a "group index" into the classification, by means of which a soil in any one group or subgroup of fine-grained materials may be rated within its own group on the basis of variations in percentages of coarse material, liquid limit, and plasticity index. This group index or "in-group" rating is determined from an empirical formula, which is derived from experience with the original PR classification. It is based on the relationships between the amount of fine-grained material (material passing a No. 200 sieve) in a soil and the liquid limit and plasticity index within the significant ranges of variation of these properties. It can be quickly determined for a given soil by two simple charts which provide a graphical solution for the group index formula. The group index of a soil is reported as a whole number in parentheses after the group or subgroup symbol, as, A-4 (5) or A-6 (12). This method of reporting automatically indicates the "edition" of the PR system which was used to classify a soil.

The classification system embodied in this committee report has been variously called the Highway Research Board system and the group index system. The writer prefers to call it the "Revised PR" system, since that is actually what it is. It is strongly recommended as a complete and comprehensive language for expressing the engineering characteristics of soil material—a

TABLE 13.—Group Classification of Highway Subgrade Materials

Procedure: With the required test data available, proceed from left to right in the following chart and the correct group will be found by the process of elimination. The first group from the left into which the test data will fit is the correct classification. All limiting test values are shown as whole numbers. If fractional numbers appear on test reports, convert them to the nearest whole number for purposes of classification.

General Classification	Granular Materials[a]			Silt-Clay Materials[b]			
Group Classification	A-1	A-3	A-2	A-4	A-5	A-6	A-7
Sieve Analysis[c]							
Sieve No. 10	....	....	....	....	....	....	....
Sieve No. 40	50 max.	51 min.	....	....	....	....	....
Sieve No. 200	25 max.	10 max.	35 max.	36 min.	36 min.	36 min.	36 min.
Characteristics of:[d]							
Liquid limit	....	....	....	40 max.	41 min.	40 max.	41 min.
Plasticity index	6 max.	nonplastic	....	10 max.	10 max.	11 min.	11 min.
Group index	....	....	4 max.	8 max.	12 max.	16 max.	20 max.
General rating as a subgrade	Excellent to good			Fair to poor			

[a] 35% or less passing sieve No. 200. In the "left to right" elimination process it is necessary to place group A-3 before group A-2; but this does not signify that group A-3 is superior to group A-2. [b] More than 35% passing sieve No. 200. [c] Percentage by weight passing sieves No. 10, No. 40, and No. 200. [d] Fraction passing sieve No. 40.

system whose terms are widely understood and recognized, which is based on long usage and experience, and which is easy to apply in day to day practice.

The following description of classification groups is extracted from the report, as are Tables 13 and 14 and Fig. 13.

TABLE 14.—Subgroups Classification of Highway Subgrade Materials

(Letter M Designates Maximum Values; Letter m Designates Minimum Values)

Procedure: With the required test data available, proceed from left to right in the following chart and the correct group will be found by the process of elimination. The first group from the left into which the test data will fit is the correct classification.

General Classification	Granular Materials[a]							Silt-Clay Materials[b]			
Groups	A-1		A-3	A-2				A-4	A-5	A-6	A-7
Subgroups	A-1-a	A-1-b	....	A-2-4	A-2-5	A-2-6	A-2-7	....	....	....	A-7-5 A-7-6
Sieve Analysis:[c]											
Sieve No. 10.......	50 M	....	....	....	....	....	....	....	....	....	....
Sieve No. 40.......	30 M	50 M	51 m	....	....	....	....	....	....	....	....
Sieve No. 200......	15 M	25 M	10 M	35 M	35 M	35 M	35 M	36 m	36 m	36 m	36 m
Characteristics of:[d]											
Liquid limit........	....		....	40 M	41 m	40 M	41 m	40 M	41 m	40 M	41 m
Plasticity index....	6 M		nonplastic	10 M	10 M	11 m	11 m	10 M	10 M	11 m	11 m[e]
Group index[f]........	0		0	0		4 M		8 M	12 M	16 M	20 M
Usual types of significant constituent materials..	Stone fragments, gravel, and sand		Fine sand	Silty or clayey gravel and sand				Silty soils		Clayey soils	
General rating as a subgrade.............	Excellent to good					Fair to poor					

[a] 35% or less passing sieve No. 200. As in Table 13, group A-3 precedes group A-2; but this does not signify that group A-3 is superior to group A-2. [b] More than 35% passing sieve No. 200. [c] Percentage by weight passing sieves No. 10, No. 40, and No. 200. [d] Fraction passing sieve No. 40. [e] The plasticity index of subgroup A-7-5 is equal to, or less than, the liquid limit −30. The plasticity index of subgroup A-7-6 is greater than the liquid limit −30. [f] The group index is computed by reference to Fig. 13 and is shown in parentheses after the group symbol; thus: A-2-6(5).

Granular Materials—Containing 35% or Less Passing a No. 200 Sieve.—

Group A-1.—The typical material of this group is a well-graded mixture of stone fragments or gravel, coarse sand, fine sand, and a nonplastic or feebly plastic soil binder. However, this group includes also stone fragments, gravel, coarse sand, volcanic cinders, etc., without soil binder.

Subgroup A-1-a includes materials consisting predominantly of stone fragments or gravel, either with or without a well-graded binder of fine material.

Subgroup A-1-b includes those materials consisting predominantly of coarse sand, either with or without a well-graded soil binder.

Group A-3.—The typical material of this group is fine beach sand or fine desert blow sand, without silty or clay fines or with a very small amount of nonplastic silt. The group includes also stream deposited mixtures of poorly graded fine sand and limited amounts of coarse sand and gravel.

Group A-2.—This group includes a wide variety of "granular" materials which are border line between the materials falling in groups A-1 and A-3 and the silt-clay materials of groups A-4, A-5, A-6, and A-7. It includes all materials containing 35% or less passing a No. 200 sieve, which cannot be classified as A-1 or A-3 due to fines content or plasticity, or both, in excess of the limitations for those groups.

Subgroups A-2-4 and A-2-5 include various granular materials containing 35% or less passing a No. 200 sieve and with a minus No. 40 portion having the characteristics of the A-4 and A-5 groups. They include such materials as gravel and coarse sand with silt content or plasticity index in excess of the limitations of group A-1, and fine sand with nonplastic silt content in excess of the limitations of group A-3.

Subgroups A-2-6 and A-2-7 include materials similar to those described under subgroups A-2-4 and A-2-5, except that the fine portion contains plastic clay having the characteristics of the A-6 or A-7 group. The approximate combined effects of plasticity indexes in excess of 10 and percentages passing a No. 200 sieve in excess of 15 are reflected by group index values of from 0 to 4.

Silt-Clay Materials—Containing More Than 35% Passing a No. 200 Sieve.—

Group A-4.—The typical material of this group is a nonplastic or moderately plastic silty soil usually having 75% or more passing a No. 200 sieve. The group includes also mixtures of fine, silty soil and up to 64% of sand and gravel retained on a No. 200 sieve. The group index values range from 1 to 8, with increasing percentages of coarse material being reflected by decreasing group index values.

Group A-5.—The typical material of this group is similar to that described under group A-4, except that it is usually of diatomaceous or micaceous character and may be highly elastic as indicated by the

high liquid limit. The group index values range from 1 to 12, with increasing values indicating the combined effect of increasing liquid limits and decreasing percentages of coarse material.

Group A-6.—The typical material of this group is a plastic clay soil usually having 75% or more passing a No. 200 sieve. The group includes also mixtures of fine clayey soil and up to 64% of sand and gravel retained on a No. 200 sieve. Materials of this group usually have a high volume change between wet and dry states. The group index values range from 1 to 16, with increasing values indicating the combined effect of increasing plasticity indexes and decreasing percentages of coarse material.

Group A-7.—The typical material of this group is similar to that described under group A-6, except that it has the high liquid limits characteristics of the A-5 group and may be elastic as well as subject to high volume change. The range of group index values is from 1 to 20, with increasing values indicating the combined effect of increasing liquid limits and plasticity indexes and decreasing percentages of coarse material.

Subgroup A-7-5 includes those materials with moderate plasticity indexes in relation to liquid limit and which may be highly elastic as well as subject to considerable volume change.

Subgroup A-7-6 includes those materials with high plasticity indexes in relation to liquid limit and which are subject to extremely high volume change.

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(24) **"Definitions of Soil Components,"** Rept. of Committee VII on Foundations and Soil Mechanics, *Civil Engineering Bulletin*, Civ. Eng. Div., Am. Soc. for Eng. Education, March, 1947.

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DISCUSSIONS

TESTS OF TIMBER STRUCTURES FROM GOLDEN GATE INTERNATIONAL EXPOSITION

Discussion

BY WALTER M. PRATT

WALTER M. PRATT,[33] Assoc. M. ASCE.—In the "Foreword" of this paper reference is made to "The need of keeping adjacent faces of members in contact * * *." The moisture content of the wood is apt to vary even after long periods of service in a structure; there may possibly be no moisture equilibrium condition. Use of moisture control might avoid many problems in wood construction. In contrast to various treatments of woods to resist moisture, so-called "fog-outlets" are available from some of the automatic sprinkler companies. These units are designed to control the humidity in a building automatically.

The Committee refers favorably to the use of plywood. Bonding strength of the glue used does not seem to be mentioned, but it might reasonably be as good as the wood in longitudinal shear, or from 100 lb per sq in. to 150 lb per sq in. The wider use of glue at surfaces of contact might be helpful, just as is the use of phenolic resin glues in laminated wood truss work, particularly with recent developments of the high-frequency electric current method of application. Casein glues might be practicable for use without special heat facilities for temperatures as low as 40° F. Surfaces of contact referred to would include splice plates, heel pads, and assemblies of web members in contact with chords.

In Fig. 28 the bottom chord joint pad has a contact of $7\frac{1}{2}$ in. by 18 in., or 135 sq in. Using a unit stress of 100 lb per sq in. on this area, the total transmitted load is 13,500 lb. Because of a possible lack of uniformity in sizing the wood, a reduction factor could be applied, but it seems likely that the stress in the split rings shown could be materially reduced, at the same time keeping the adjacent surfaces in contact. Without adhesives, the friction between wood

NOTE.—This paper by a Committee of the San Francisco (Calif.) Section, ASCE, on Timber Test Program was published in May, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1947, by James H. Carr, Jr., Howard J. Hansen, and E. G. Stern; November, 1947, by Melvin W. Jackson, and Charles Mackintosh; and January, 1948, by Sidney Novick, Frank J. Hanrahan, and Walter J. Ryan.

[33] Mgr., Pratt Roof Truss Co., Detroit, Mich.

surfaces in contact can be a factor in carrying loads, but it is often lost through wood shrinkage, and its amount is doubtful.

The background of these tests is such that all the truss work shown is of "ordinary" construction and not "slow burning." A slow burning truss can be defined roughly as one in which no single member has an area of less than 56 sq in. Use of the authors' data on plywood for gussets suggests possibilities for future truss designs where greater resistance to fire is desired through slow burning construction.

Several joints tested involved the use of "split rings," and in several cases the wood cores sheared. The results indicate the value of tests on full-size trusses, compared with laboratory tests often made. The tension-across-grain phase is very properly emphasized. A hollow ring dowel, and any round dowel, must necessarily produce a splitting action. There is no constant angle to the grain in a round dowel. The force in the dowel may be at an angle to the grain, but the bearing of the ring dowel on the wood is never at a single angle—rather it is at many angles.

Referring to Fig. 36, it is also possible that the core within a ring may receive load before the exterior bearing of the ring, producing an extra "punching shear." The wood core may or may not take up one half the ring load. It is customary, but not necessarily correct, to assume that each half of the ring takes one half of the load. With a ring 4 in. in inside diameter having a center bolt $\frac{3}{4}$ in. in diameter, the core shear area is 12.12 (say, 12) sq in. At 150 lb per sq in. its load transmitting value would be 1,800 lb. Let the value for this ring be 6,400 lb at 0° to the wood grain and let the value at 90° be 4,500 lb. If the core is working independently, and is limited in capacity by horizontal shear, the opposing halves of the ring must carry 4,600 lb and 2,700 lb, respectively. Dividing by the projected area of $4 \times 1 \times \frac{1}{2}$ or 2 sq in., the resultant unit stresses would be 2,300 lb per sq in. and 1,350 lb per sq in., respectively—even if the bearing were truly at 0° and 90°, respectively. The allowable stresses for these rings, given in "III. Nomenclature: Explanation of Terms and Abbreviations," paragraph 3, can be compared with the design stresses described under the same heading (paragraph 1).

If a hollow square dowel or a hollow rectangular dowel were available, the core could be analyzed on the basis of the allowable stresses mentioned in the "Nomenclature." Thus, a hollow square dowel, 4 in. by 4 in. by 1 in., at 90° to the grain would develop a resistance of $2 \times 4 \times \frac{1}{2} \times 350 = 1{,}400$ lb, compared with the 4,500 lb previously stated for the 4-in. split ring.

The round dowel is used partly because a convenient turning tool is available; a hollow square dowel could be tooled with a routing machine. As the authors state, workmanship is a very important factor. There is also the possibility that round wood cores shrink to something approaching an ellipse. From an economic standpoint, structural use from the core is desirable, but the dowel design should have balance, and the same factor of safety could well be used for ring cores as is demanded for the remainder of the structure.

To approach the phase of tension across the grain from a different viewpoint, a round dowel can be compared with a square one. If a hollow square dowel, 4 in by 4 in. by 1 in., has a value of only 1,400 lb at right angles to the grain, and

if a 4-in. standard split ring has a value of 4,500 lb under similar conditions, it can be assumed that a ring-type dowel never acts at any one angle to the grain. Consider the wedging and splitting action of such a 4-in. square dowel set at 45° to the grain. It would have a theoretical bearing area of $2 \times 4 \times \frac{1}{2} \times 1.41 = 5.6$ sq in. At an allowable stress of 540 lb per sq in. (determined by the Hankinson formula), this area will transmit a stress of 3,050 lb, which compares with 4,500 lb—the value of a 4-in. split ring at a 45° bearing angle. It seems likely then that tension across the grain is inherent in the ring-type dowel, and that forces that tend to separate the fibers of the wood are set up.

Regarding reentrant cuts in wood—some may be eliminated. If a joint assembly needs a cluster of ring dowels, the web members could at times be balanced around resultant force lines as shown in Fig. 88, using only one row of dowels, as is shown in the bottom chord.

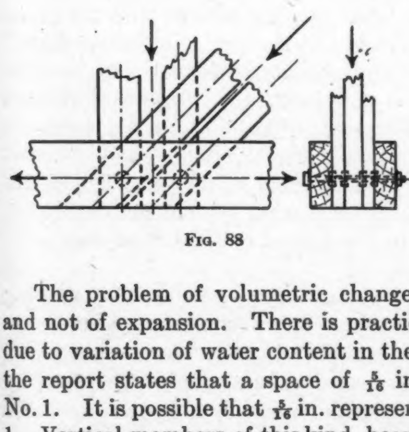

Fig. 88

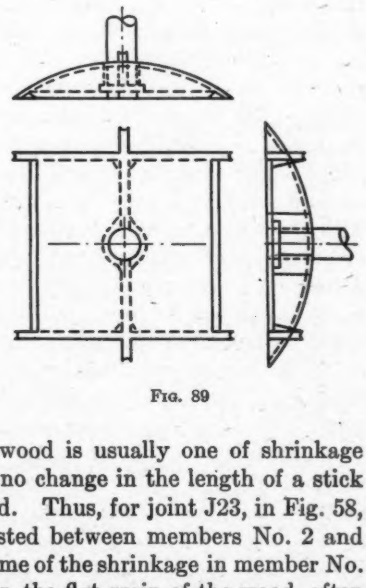

Fig. 89

The problem of volumetric changes in wood is usually one of shrinkage and not of expansion. There is practically no change in the length of a stick due to variation of water content in the wood. Thus, for joint J23, in Fig. 58, the report states that a space of $\frac{5}{16}$ in. existed between members No. 2 and No. 1. It is possible that $\frac{5}{16}$ in. represents some of the shrinkage in member No. 1. Vertical members of this kind, bearing on the flat grain of the wood, often show such a separation, and the authors rightly recommend that such details be avoided. The use of free body diagrams will often assist the designer to avoid such bearings. Thus, if the vertical members in Fig. 88 were fastened on the outside of the chord, the angle to the grain on chord would be 90°. Placed as shown, the diagonal is the only member working at an angle to the grain.

However, there is the possibility in a joint assembly that different members shrink at different rates and at different angles. Splice pads may have different water contents than the member joined and they may shrink at a different rate than the member, thereby causing possible secondary stresses, particularly if more than one row of bolts or connectors is used. Each member may shrink in both width and thickness, the shrinkage usually being about one third less radial to the annual growth rings than tangential to them. Depending on how

the lumber is cut from the log, shrinkage may be unbalanced and cause warping and twisting. Lumber is certainly an intriguing material for the designer. It would seem best, as often as possible, to keep wood connectors on the center lines of joined pieces, and to avoid clusters of off-set connectors.

The report stresses the need for new thought and improvements on timber connectors. It indicates the desirability of a type of connector that can be analyzed readily and also the desirability of reducing flat grain bearings, of providing for wood shrinkage, and of preventing the overstepping tolerances by manual labor. In order to avoid the complications in stress analysis of ring dowels, the use of plane surface contacts instead of rounded contacts would greatly simplify analysis. The report definitely uses, in making comparisons, "connector allowable load." Such loads were apparently developed from tests by others. Undoubtedly, in a great deal of construction, hollow ring dowels have been used with satisfactory results, but their continued wide application, even at reduced values, would still leave many engineers hoping for an improved connector after considering all the factors—safety, rational basis of design, feasibility, and economy. Perhaps one of the writer's own designs (Fig. 89) might furnish some constructive thought along these lines, at least as a new angle of approach. The unit shown would have two arcuate wings placed at right angles to the grain, and one wing parallel with the grain, all connected by a web. It could be used in the manner of a "shear plate," with the whole connection computed as a pin-connected joint. The concave or beveled edges shown might be effective in reducing the effects of shirnkage. A limiting factor might be the single shear value of the bolt, or the resistance value of the bolt to bending. The longitudinal shear value of the wood core could be regulated by the distance between the pair of flanges. The single center fin is entirely subtended by the bolt. Whatever type of connector may be devised, the underlying difficulties are the lack of constancy of volume in wood, and its nonhomogeneous character.

Concerning the workmanship—some play must likely be allowed in making wood fittings. Possibly the use of cementitious fillers (similar in character to plastic wood), powdered metal with an asphaltic binder, thermoplastics, or metal shims could be considered. Tolerances in workmanship possibly may be overcome by means other than providing flexibility in a dowel.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

EXPERIENCES WITH PREDETERMINING PILE LENGTHS

Discussion

By A. W. Earl

A. W. Earl,[3] M. ASCE.—The method for determining the proper lengths for foundation piles as developed by the author and his associate, Trent R. Dames, M. ASCE, has been used with success on several jobs with which the writer is familiar, to the extent of several thousand piles. From acquaintance thus gained, the method is believed to be the best and most practical so far proposed in that it provides piles no longer than necessary for safety and takes into account the effect of settlement in the upper layers of the soil.

In the absence of any better procedure for the past fifty or sixty years it has been usual to drive to a bearing capacity as determined by the *Engineering News* formula, or some formula of similar type. In spite of the fact that such formulas leave out many important factors and are supposedly quite limited in their application, the results, on the whole, have been satisfactory in so far as safety is concerned, and the percentage of failures has been exceedingly small. However, although piles driven to a bearing capacity determined by the *Engineering News* formula are generally of safe length, it is probable that in most cases they are considerably too long. In these days of high prices, even a small saving in length per pile on a large job means a large saving in money. Another serious drawback to the use of a driving formula is that it gives no indication of the ultimate capacity of the pile after settlement has taken place. Engineers realizing these and other inadequacies have attempted more rational methods of procedure, but so far as is known, none of these methods, aside from the one described by Mr. Moore, has been very successful.

The method of Messrs. Dames and Moore for determining pile lengths takes into account, more or less rigorously, the various factors that make up the bearing capacity of a pile and, at the same time, makes possible an evaluation of the loads that subsidence of upper strata will impose on the pile. It therefore does the important things that such a method should do—and it does them very well. However, anyone who has delved into soil mechanics

Note.—This paper by William W. Moore was published in November, 1947, *Proceedings*.

[3] Cons. Civ. Engr., San Francisco, Calif.

literature or has followed the discussions on the subject in the *Proceedings* of the Society can readily imagine that a number of very erudite objections might be raised to such a simple, straightforward method of attack.

One such objection might be that no account has been taken of deflections, both of the pile with reference to the soil and of the various strata with regard to each other, and how these deflections affect the distribution of skin friction on the pile. A question might also be raised as to whether the firmer strata might not have to supply more than their share of resistance because the deflection of the pile might not be enough to mobilize the full resisting forces in the softer strata. However, in view of the differences in subsurface conditions usually found from place to place at a site, too theoretical a viewpoint might lead to a result no closer to the facts than the simpler approach, and the fact remains that any method for determining lengths of piles must stand or fall on its service record and the results of tests in the field. The Dames and Moore method is no exception, and this paper is therefore of particular interest in that it shows the correspondence of calculated values and test values for a variety of soils and types of piles.

It is outside the scope of the author's paper to discuss the reduction in load capacity per pile for piles in groups, but it may be well to suggest that it is something that should be taken into account—even more carefully with the Dames and Moore method than when driving to bearing capacity as determined by the *Engineering News* formula, because in general the piles will be shorter and their surplus capacity will be less.

Pile driving can never be reduced to exact rules and the knowledge gained from the logs of test holes is extremely useful in making such adjustments in driving as are frequently necessary. It should be stressed that one of the great virtues of the method is that it necessitates taking borings. This virtue should be cherished. In addition, the number of borings should be generous, otherwise the contours of penetration are mere guesses that lead to endless trouble in the field.

With no background of service records or tests, it required a certain amount of courage on the part of Messrs. Dames and Moore to sponsor this new method and assume the responsibility such sponsorship entailed. That the method has stood the test, as evidenced by the data presented, must be very gratifying to them.

Correction for *Transactions*.—In November, 1947, *Proceedings*, on page 1354, the curve marked "Natural soil pressure" should terminate at El. − 130 with a compressive stress of 4.9 kips per sq ft.

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DISCUSSIONS

EXPERIMENTAL DETERMINATION OF VIBRATION CHARACTERISTICS OF STRUCTURES

Discussion

BY ALEXANDER KLEMIN

ALEXANDER KLEMIN,[3] ESQ.—This paper is in the best traditions of scientific instrumentation. Although the theory is soundly conceived, it appears complex at first. However, it no longer appears complex if one checks back against the Rayleigh theory, and the analyzer itself is simple (though ingenious) in construction and operation.

The investigation of flutter and vibration is a vital and difficult element of airplane design. Before any calculations can be made, a number of tests are necessary—tests concerned with the natural frequencies of different parts of the airplane, the location of nodes and axes, and many other pertinent data. Frequencies are formed by setting up vibrations, most often by applying a periodic force and finding a resonant condition. Electro-mechanic oscillators of some complexity are used. The instrumentation is not too portable, and there is some hazard in setting up resonance. Mr. Loring's instrument appears superior in portability, in simplicity of operation, and in absence of any hazard. It is an added advantage that the instrument is so light that it should be possible to produce flexural vibration without torsional vibration. The writer can vouch for its great utility in the aircraft field.

Corrections for *Transactions:* In December, 1947, *Proceedings,* on page 1460, Eq. 8*b*, insert "$=$" after "$(m_i)_k$"; on page 1464, in Eq. 26 (second line), change "e^{ov}" to "e^{-ov}" and, in Eq. 28 (third line), insert a bracket between the "$+$" and "$\frac{1}{2}$" and, in Eq. 28 (last line), insert "r" under the radical sign; on page 1467, in Eq. 42, change "$- \frac{2 \log_e \theta}{g_o}$" to "$\times \frac{- 2 \log_e \theta}{g_o}$"; and, on page 1472, line 19, the last numeral in the numerator should read 55.28 and, following the last line, insert

ACKNOWLEDGMENT

"Patents are pending on certain instruments based on the ideas advanced in this paper. Nevertheless, the writer offers the results of his experience, together with patent rights, for the free use of the profession. It is expected merely that suitable acknowledgment or recognition will be made by those who use the patentable principles involved."

NOTE.—This paper by Samuel J. Loring was published in December, 1947, *Proceedings.*

[3] Greenwich, Conn.

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DISCUSSIONS

LEAST SQUARES ADJUSTMENT OF TRIANGULATION NET BETWEEN GEODETIC STATIONS

Discussion

BY JULIUS L. SPEERT, C. H. NEY, H. W. HEMPLE, AND LANSING G. SIMMONS

JULIUS L. SPEERT,[5] M. ASCE.—A strong case is presented in this paper for the adjustment of local triangulation nets to the basic geodetic datum when ties are available at two or more points. The subject is particularly timely because of two parallel but independent developments in recent years. The first of these is the continued expansion and the breakdown of the basic triangulation net by the establishment of a large number of closely spaced geodetic stations. This has brought geodetic control into many regions that previously had been without it. The second important development was the introduction and acceptance of the state plane coordinate systems.

These innovations make it possible to express geodetic data in terms of simple x-coordinates and y-coordinates that are convenient for use by all local surveyors. The old excuse that geodetic data are too complicated for purely local use is no longer applicable. To conform with the trend of bringing the strength of geodetic control within the scope of the local surveyor's use, the methods of geodetic computation must be simplified in their application to small local survey nets computed on a plane coordinate grid. The paper presents a worthy contribution in that direction.

It is appropriate to stress that the method of adjustment proposed by the authors is essentially the same as that recommended and used for many years by the United States Coast and Geodetic Survey and described in detail in its publications.[6,7] The principal improvement made by the authors is the adaptation of the method to computation in plane coordinates and their demonstration of its true simplicity.

NOTE.—This paper by E. F. Coddington and O. C. J. Marshall was published in September, 1947, *Proceedings*.

[5] Topographic Engr., U. S. Geological Survey, Washington, D. C.

[6] "Application of the Theory of Least Squares to the Adjustment of Triangulation," *Special Publication No. 28*, U. S. Coast and Geodetic Survey, 1915.

[7] "Manual of Triangulation Computation and Adjustment," *Special Publication No. 138*, U. S. Coast and Geodetic Survey, 1934.

The problem of adjusting a local net to the basic geodetic datum will fall naturally into one of two cases—according to whether the local net was established before or after the ties to the basic datum became available. Each case is subject to its own particular type of treatment.

In the first case, where the local net was established first, and properly adjusted internally before the geodetic datum was extended into the area, the problem becomes one of readjustment of datum. Perhaps the simplest method of datum readjustment is by the use of "isodiffs." This method has been described in its application to geodetic triangulation,[8] but the appropriate simplifications should be readily apparent to any engineer who is interested in using it on a plane coordinate system. The method is especially advantageous when either funds or time for complete recomputation and adjustment are not readily available. If the original adjustment was made by the least squares method of condition equations, and if the original computations are available to the computers, the addition of equations for the new conditions imposed by the ties (as recommended by the authors) offers an excellent opportunity to complete the adjustment without repeating the bulk of the original computations. Both methods take full advantage of the previous computations and adjustment.

In the second case, when ties to the basic geodetic datum are available at the time the local net is established, the original adjustment should, of course, be based on that datum. In this case the computer has greater latitude in his choice of method. The two basic methods of geodetic adjustment—condition equations and variation of coordinates—are both available. Both lend themselves readily to considerable simplification in their application to a small local net on a plane coordinate system.

The authors have presented an excellent description of the condition equation method. However, for this particular type of survey net the coordinate method is usually so far superior to the condition method as to economy of labor that it would appear to justify far more consideration than it has received. A detailed description of its application to a plane coordinate system was published as far back as 1935.[9] In this method there will always be just two normal equations for each new station established, regardless of the complexity of the net, or of the amount of overlapping of its component parts. Thus, the adjustment in Fig. 1 would require the solution of only eight simultaneous equations as compared to the sixteen equations of the condition method; and the adjustment in Fig. 4 would require the solution of ten equations instead of twenty one. The addition of further ties, check observations, or other complexities to the problem merely increases the advantages of the coordinate method over the condition method.

C. H. Ney,[10] Esq.—An interesting and practicable method of adjusting a local triangulation net between two fixed points, that have been established

[8] "Readjustment of Triangulation Datum," by Julius L. Speert, *Transactions*, ASCE, Vol. 103, 1938, p. 1002.

[9] "Triangulation Adjustments in Plane Coordinate Systems," by R. M. Wilson, *Engineering News-Record*, October 3, 1935, p. 459.

[10] Chf., Triangulation Adjustments Div., Geodetic Survey of Canada, Dept. of Mines and Resources, Ottawa, Ont., Canada.

with a higher degree of accuracy, has been outlined by the authors. As the positions of the survey stations are defined in terms of rectangular coordinates, surveyors and city engineers who have occasion to make use of this type of adjustment will be spared the necessity of familiarizing themselves with the computation of geodetic latitudes and longitudes.

With one of the fixed stations taken as the origin, the latitudes and departures of the other triangulation stations are determined. A traverse, composed of selected triangle sides joining the fixed points, is used in the formation of the condition equations of closure. For the determination of the absolute terms of these equations, the latitude and departure of the second fixed point must be computed through the traverse. The accuracy of the triangulation adjustment hinges upon the correct determination of these values, which, in reality, define the azimuth and length of the polar ray joining the two fixed points.

The authors compute the required latitude and departure of the second fixed point by plane trigonometry on the premise that the survey is of such limited extent that no error will result from neglecting spherical excess and the curvature of the earth. However, it should be kept in mind that there is a definite limit to the physical extent of the local survey beyond which the authors' methods of adjustment will not be applicable. In this discussion an attempt will be made to formulate a criterion for the determination of this limit.

In 1928, W. M. Tobey, then Assistant Director of the Geodetic Survey of Canada, issued a bulletin[11] outlining a method whereby a traverse having courses of almost unlimited length may be treated in terms of latitudes and departures by an application of Legendre's theorem to the polar triangles formed by lines joining each traverse station with the origin.

Fig. 6(*a*) shows a traverse on the spheroid, and Fig. 6(*b*) shows a corresponding "Legendrian" traverse on a plane where the measured deflection angles, $\beta_2, \beta_3, \beta_4, \cdots, \beta_n$, are increased by $\frac{\epsilon_2}{3}, \frac{\epsilon_2 + \epsilon_3}{3}, \frac{\epsilon_3 + \epsilon_4}{3}, \cdots, \frac{\epsilon_{n-1} + \epsilon_n}{n-1}$, and where the distances, $t_1, t_2, t_3, \cdots, t_n$, of the Legendrian traverse are identical with the corresponding values determined from actual measurement on the earth's surface. It is seen that the deflection angle at any point of the corresponding Legendrian traverse is greater than the corresponding deflection angle of the actual traverse by one third of the sum of the spherical excesses of the polar triangles adjacent to the polar ray.

The bearings, $b_1, b_2, b_3, \cdots, b_n$, of the courses of the Legendrian traverse are

$$b_1 = b_{1a} \qquad (17a)$$

$$b_2 = b_{2a} + \frac{\epsilon_2}{3} \qquad (17b)$$

$$b_3 = b_{3a} + \frac{2}{3}(\epsilon_2 + \epsilon_3) - \frac{\epsilon_3}{3} \qquad (17c)$$

[11] "The Conversion of Latitudes and Departures of a Traverse to Geodetic Differences of Latitude and Longitude," by W. M. Tobey, *Publication No. 25*, Geodetic Survey of Canada, Ottawa, Ont., 1928.

and

$$b_n = b_{na} + \frac{2}{3}(\epsilon_2\epsilon + {}_3 + \cdots + \epsilon_n) - \frac{\epsilon_n}{3} \qquad \text{(17}d\text{)}$$

in which b_{1a}, b_{2a}, $\cdots$, b_{na} are approximate bearings of the measured courses referred to the meridian through the origin, and computed from the north, as follows:

$$b_{1a} = \beta_1 \qquad \text{(18}a\text{)}$$

$$b_{2a} = \beta_1 + \beta_2 \qquad \text{(18}b\text{)}$$

and

$$b_{na} = \beta_1 + \beta_2 + \beta_3 + \cdots + \beta_n \qquad \text{(18}c\text{)}$$

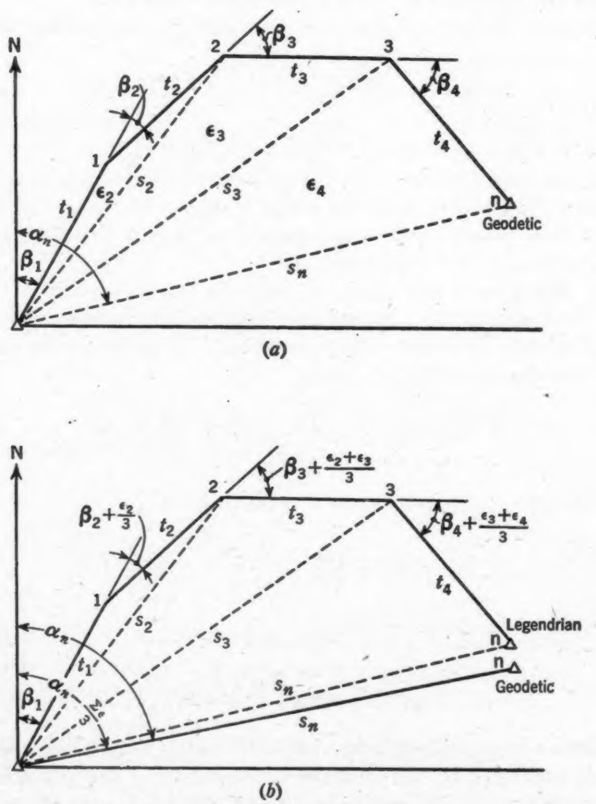

Fig. 6

The Legendrian latitudes and departures for the nth-station for the traverse are

$$L_n = L_{na} - [\tfrac{2}{3}(\epsilon_2 + \epsilon_3 + \epsilon_n) - \tfrac{1}{3}\epsilon_n]\, D_{na} \sin 1'' \qquad \text{(19}a\text{)}$$

and

$$D_n = D_{na} + [\tfrac{2}{3}(\epsilon_2 + \epsilon_3 + \epsilon_n) - \tfrac{1}{3}\epsilon_n] L_{na} \sin 1'' \qquad \text{(19}b\text{)}$$

in which L_n and D_n are the Legendrian values of the latitudes and departures,

respectively, and

$$L_{na} = t \cos b_{na} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20a)$$

and

$$D_{na} = \text{t} \sin b_{na} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20b)$$

are the approximate values as computed in ordinary traverse reductions. The addition of the corrective terms to the approximate latitudes and departures allows for reduction of traverses up to 120 miles in length with geodetic precision. Before the length and azimuth of a polar ray that extends from the origin to any one of the traverse stations may be determined, it is necessary to calculate the spherical excesses of the polar triangles.

The spherical excesses of the polar triangles are determined from

$$\epsilon''_n = \frac{D_{na}\, L_{(n-1)a} - D_{(n-1)a}\, L_{na}}{2\, R\, N \sin 1''} + K \ldots\ldots\ldots\ldots\ldots (21)$$

in which K is a second order term of negligible magnitude. It is seen that, in calculating the excesses, the approximate values of the latitudes and departures may be used. Attention must be given to the correct signs of the latitudes and departures—points north of the origin have a plus latitude and points east of the origin have a plus departure.

Having determined the corrected latitude and departure of the terminal station of the traverse, the azimuth and length of the polar ray from the origin to this station may be found. If ψ represents the angle on the plane between the initial meridian and the polar ray,

$$\tan \psi = \frac{D_n}{L_n} = \frac{D_a + dD}{L_a + dL} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

The true azimuth corresponding to ψ is given by

$$\alpha_n = \psi_n + \left(\frac{\epsilon_2}{3} + \frac{\epsilon_3}{3} + \cdots + \frac{\epsilon_n}{3} \right) \ldots\ldots\ldots\ldots\ldots (23)$$

and the length of the polar ray is

$$S_n = \frac{D_n}{\sin \psi_n} = \frac{L_n}{\cos \psi_n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

In adjusting local triangulation between widely separated fixed points by the methods suggested by the authors, the accuracy of the adjustment may be increased by taking into consideration the Legendrian corrections to the latitudes and departures.

Example 1.—Taking the length of the line 1–2 and the adjusted angles (figural adjustment only) pertaining to the authors' first numerical example, the lines 1–2, 2–3, 3–6, 6–7, and 7–n, were worked out and treated as a traverse (Fig. 7). Computed in the usual manner, the azimuth and length of the polar ray 1–n are found to be $\alpha_{1-n} = 34° \, 32' \, 50.65''$ and $\log S_{1-n} = 4.0542039$ m. The Legendrian reduction gives, for the same line, $\alpha_{1-n} = 34° \, 32' \, 50.72''$ and

log S_{1-n} = 4.0542039 m. For a survey of limited extent, as typified by this example, it is evident that no increased accuracy of practical importance is obtained by the application of the Legendrian principle.

Example 2.—To show the application of the Tobey method[11] for finding the azimuth and length of a long polar ray, the traverse of the preceding example (Fig. 7) has been recomputed using the original deflection angles. The length of each course has been converted to meters, however, and multiplied by 6. As this theoretical traverse extends over an area 35 miles from

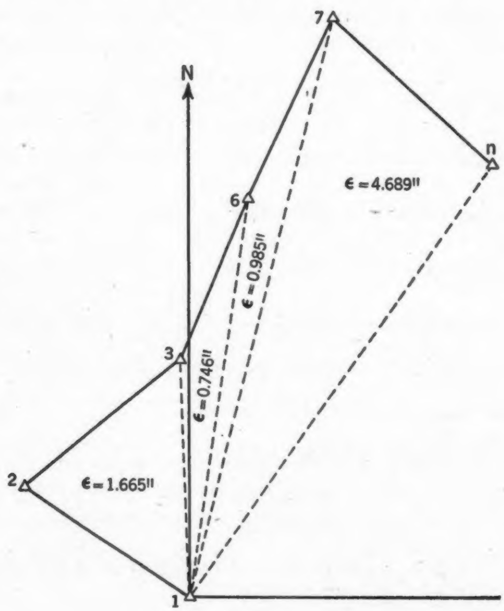

FIG. 7

north to south and 38 miles from east to west, it is sufficiently large to benefit by the Legendrian refinement in making the computations.

Fundamental data of the expanded traverse are as follows:

(a) Geographical position of the origin, Station 1, $\phi = 45° \; 00' \; 00.000''$ (assumed) and $\lambda = 75° \; 00' \; 00.000''$ (assumed);

(b) $\alpha_{1-2} = 304° \; 27' \; 16.53''$ (assumed) clockwise from the north;

(c) Measured lengths of courses—

Course	Log of length, in meters
1–2	4.4190809
2–3	4.4175670
3–6	4.3508778
6–7	4.3974938
7–n	4.4385440

(*d*) Measured deflection angles—

$$\begin{aligned}
\beta_1 &= +\ 304^\circ\ 27'\ 16.53'' \\
\beta_2 &= +\ 106^\circ\ 58'\ 15.95'' \\
\beta_3 &= -\ \ \ 28^\circ\ 44'\ 37.70'' \\
\beta_6 &= +\ \ \ \ \ 2^\circ\ 46'\ 42.55'' \\
\beta_7 &= +\ 106^\circ\ 40'\ 29.75''
\end{aligned}$$

The various steps of the reduction are given in detail hereinafter.—

a. The determination of the approximate bearings of the courses (reckoned clockwise from the north and referred to the meridian through Station 1) is as follows:

$$\begin{array}{lcr}
\text{Azimuth 1–2} & = & 304^\circ\ 27'\ 16.53'' \\
\beta_2 & = & \underline{106^\circ\ 58'\ 15.95''} \\
b_{2a}\ \text{(approximate bearing 2–3)} & = & 51^\circ\ 25'\ 32.48'' \\
\beta_3 & = & \underline{-\ 28^\circ\ 44'\ 37.70''} \\
b_{3a}\ \text{(approximate bearing 3–6)} & = & 22^\circ\ 40'\ 54.78'' \\
\beta_6 & = & 2^\circ\ 46'\ 42.55'' \\
b_{4a}\ \text{(approximate bearing 6–7)} & = & \underline{25^\circ\ 27'\ 37.33''} \\
\beta_7 & = & \underline{106^\circ\ 40'\ 29.75''} \\
b_{5a}\ \text{(approximate bearing 7–n)} & = & 132^\circ\ 08'\ 07.08''
\end{array}$$

Steps (*b*), (*c*), (*d*), and (*e*), are shown in Tables 9 to 12, respectively, in which all measurements are given in meters.

f. Azimuth and length of polar ray 1–n—

$$\operatorname{Tan} \psi = \frac{D}{L} = \frac{38{,}548.32}{55{,}988.26}$$

$$\begin{aligned}
\operatorname{Log} \tan \psi &= 4.58600546 - 4.74809697 = 9.83790849 \\
\psi &= 34^\circ\ 32'\ 51.68'' \\
\Sigma(\epsilon/3) &= \underline{+\ \ \ \ \ \ \ \ \ \ 2.69''} \\
\alpha_{1-n} \quad &= 34^\circ\ 32'\ 54.37''
\end{aligned}$$

$$\begin{aligned}
\operatorname{Log} L_n &= 4.74809697 \\
\operatorname{Cos} \psi &= \underline{9.91574505} \\
\operatorname{Log} S &= 4.83235192 \\
S &= 67{,}975.32 \text{ m}
\end{aligned}$$

Thus, for the polar ray 1–n,

$$\begin{aligned}
\alpha_{1-n} &= 34^\circ\ 32'\ 54.37'' \\
\operatorname{Log} S_{1-n} &= 4.8323519 \\
S &= 67{,}975.42 \text{ m}
\end{aligned}$$

Had the values been computed without applying the Legendrian corrections, the result would have been

$$\begin{aligned}
\alpha_{1-n} &= 34^\circ\ 32'\ 50.53'' \\
\operatorname{Log} S_{1-n} &= 4.8323549 \\
S &= 67{,}975.89 \text{ m}
\end{aligned}$$

TABLE 9.—APPROXIMATE LATITUDES AND DEPARTURES OF COURSES

Course or station	Bearing[a]	Logarithm of: t[(b)]	cos b_a	sin b_a	Latitude[a]	Departure[a]	Latitude[a,b] L_a	Departure[a,b] D_a
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1-2	304° 27′ 16.53″	4.4190809	9.7526268	9.9162301	4.1717077	4.3353110	14,849.36	−21,642.68
2-3	51° 25′ 32.48″	4.4175670	9.7948570	9.8930957	4.2124240	4.3106627	16,308.86	20,448.56
3-6	22° 40′ 54.78″	4.3508778	9.9650417	9.5861532	4.3159195	3.9370310	20,697.58	8,650.29
6-7	25° 27′ 37.33″	4.3974938	9.9556314	9.6333540	4.3531252	4.0308478	22,548.89	10,736.13
7-n	132° 08′ 07.08″	4.4385440	9.8266472	9.8701478	4.2651912	4.3086918	−18,415.83	20,355.97
n	....	....	....	....	....	....	55,988.86	38,548.27

[a] Approximate. [b] Measured in meters.

TABLE 10.—COMPUTATION

Station	Latitude (m)	Departure (m)	Logarithm: Latitude	Logarithm: Departure[c]	$L_m D_{m+1}$
(1)	(2)	(3)	(4)	(5)	(6)
1	0.00	0.00			
2	14,849.36	−21,642.68	4.1717072	4.3353106	
3	31,158.22	− 1,194.12	4.4935729	3.0770480	− 17,731,893
6	51,855.80	+ 7,456.17	4.7147972	3.8725176	+ 232,322,069
7	74,404.69	+18,192.30	4.8716002	4.2598876	+ 943,375,906
n	55,988.86	+38,548.27	4.7481018	4.5860055	+2,868,175,414

[c] m =

TABLE 11.—EVALUATION OF ϵ-CORRECTIONS TO APPROXIMATE BEARINGS

Polar triangle	ϵ (″)	$\epsilon/3$ (″)	$2\epsilon/3$ (″)	$\Sigma\frac{2\epsilon}{3}$ (″)	Corrections to Approximate Bearings: db = Col. 5 − Col. 3 (″)	Log db	Course
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1-2-3	1.6646	0.5549	1.1097	1.1097	+0.5548	9.74414	2-3
1-3-6	0.7459	0.2486	0.4973	1.6070	+1.3584	0.13303	3-6
1-6-7	0.9851	0.3284	0.6567	2.2637	+1.9353	0.28675	6-7
1-7-n	4.6890	1.5630	3.1260	5.3897	+3.8267	0.58282	7-n
		2.6949					

TABLE 12.—LEGENDRIAN

Course	b_a	Log t (m)	db (″)	Log db	Sin 1″	$L_a = t \cos b_a$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1-2	304° 27′ 16.53″	4.4190809				14,849.36
2-3	51° 25′ 32.48″	4.4175670	0.5548	9.74414	4.6855748	16,308.86
3-6	22° 40′ 54.78″	4.3508778	1.3584	0.13303	4.6855748	20,697.58
6-7	25° 27′ 37.33″	4.3974938	1.9353	0.28675	4.6855748	22,548.89
7-n	132° 08′ 07.08″	4.4385440	+3.8267	0.58282	4.6855748	−18,415.83

In general, in adjusting local triangulation nets between fixed primary stations by the methods suggested by the authors, it is desirable to have a criterion indicating the extent of the survey which may be accurately reduced. From the foregoing, it is evident that the sum of the spherical excesses of the polar triangles involved in the survey provides this criterion. From the application of the Legendrian principle it can be shown that

$$\alpha_{1-n} = \beta_1 + \beta_2 + \beta_3 + \cdots + \beta_0 - 180° + \Sigma(\epsilon'') \qquad (25)$$

in which $\Sigma(\epsilon'')$ is the sum of the spherical excesses of the polar triangles. If $\Sigma(\epsilon'') = 0$,

$$\alpha_{1-n} = \beta_1 + \beta_2 + \cdots + \beta_0 - 180° \qquad (26)$$

OF SPHERICAL EXCESS

$L_{m+1}\,D_m$ (7)	Col. 6 − Col. 7: Value (8)	Col. 6 − Col. 7: Log (9)	Log $m^{(a)}$ (10)	Spherical excess (11)
− 674,347,194	+ 656,615,300	8.81731	1.40400	1.6646
− 61,922,024	+ 294,244,093	8.46870	1.40400	0.7459
+ 554,766,175	+ 388,599,731	8.58950	1.40400	0.9851
+1,018,566,501	+1,849,608,912	9,26708	1.40400	4.6890

$\dfrac{1}{2\,R\,N \sin 1''}$.

and the azimuth and the length of the polar ray joining the origin with the terminal station may be deduced with geodetic accuracy from the approximate latitudes and departures. The approximate value of ϵ for each polar triangle may be quickly deduced by scaling the area of each triangle, converting the area to square miles, and dividing by 76.5. Remembering that the Legendrian corrections to the latitudes and departures of the traverse courses are

$$dL_{\text{Leg}} = t \sin b_a \, db \sin 1'' \qquad (27a)$$

and

$$dD_{\text{Leg}} = t \cos b_a \, db \sin 1'' \qquad (27b)$$

LATITUDES AND DEPARTURES

Log sin b_a (8)	Latitudes: dL (m) (9)	Latitudes: $L_a + dL$ (m) (10)	$D_a = t \sin b_a$ (11)	Log cos b_a (12)	Departures: dD (13)	Departures: $D_a + dD$ (14)
9.9162301	0.000	14,849.36	−21,642.68	9.7526268		−21,642.68
9.8930957	0.0550	16,308.80	+20,448.56	9.7948567	+0.0439	20,448.60
9.5861531	0.0570	20,697.52	8,650.29	9.9650417	+0.1363	8,650.43
9.6333539	0.1007	22,548.79	10,736.13	9.9556314	+0.2116	10,736.34
9.8701478	0.3776	−18,416.21	20,355.97	9.8266472	−0.3417	20,355.63
		55,988.27				38,548.32

respectively, and that

$$\tan \psi = \frac{\Sigma(D_a + dD_{\text{Leg}})}{\Sigma(L_a + dL_{\text{Leg}})} \qquad (28)$$

the effect of the spherical excesses on the azimuth and length of the computed polar ray may be readily determined. A cursory examination in this manner will show whether the desired accuracy may be attained without the application of Legendre's theorem.

H. W. Hemple,[12] M. ASCE.—One of several methods which may be used in adjusting local triangulation surveys to the federal geodetic triangulation network is illustrated in this paper. As the density of coverage of the federal net of control established by the United States Coast and Geodetic Survey (U.S.C. and G.S.) increases, more general utilization of the stations established thereon will be made to control supplemental surveys. Connections of local surveys to the federal triangulation net will make certain that these will be in their correct relative position with respect to other surveys so connected; and also will give assurance that the points established on the local survey can be re-established at any time, should the original markings be lost. The method of adjustment discussed by Professors Coddington and Marshall brings to the attention of engineers and surveyors one system by which their own surveys may be adjusted to agree with the over-all federal net.

The U.S.C. and G.S. has completed in excess of 100,000 miles of first order and second order triangulation, forming a network of control spanning the entire United States. In addition to the main scheme stations, there are thousands of supplemental points located on these arcs. The positions of all points are expressed on the North American Datum of 1927. The national survey nets on Mexico and Canada are also related to this same datum, as is the triangulation extending into Alaska, to the tip of the Aleutian Islands, and to the Bering Strait. Since all triangulation stations on the national nets of North America are on a common datum, their locations are definitely established; and all such survey data are expressed in their correct relationship to one another. For any one station, the geographic position will satisfy only that particular location.

At certain intervals a measured base line is inserted into the scheme of triangulation. These base lines are measured with invar tapes, which have been standardized at the United States Bureau of Standards. Measurements are made with a definite tension, and temperature readings are taken at each tape length. At least three tapes are used, and measurements are made backward and forward. Checks are such that the distance is determined with an accuracy so that the actual error must not exceed 1/300,000 for first order triangulation.

The check on distances between base lines as carried through the triangulation must be 1/25,000 or better. In actual practice the average check is of the order of 1/80,000 for the 100,000 miles of triangulation in the United States. Azimuth control is provided by Laplace azimuths spaced at regular intervals. Observed astronomic azimuths are corrected for the effect of the deflection of

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the vertical through the use of precise longitude observations and are thereby reduced to true geodetic azimuths. Surveyors and engineers should have no hesitancy concerning the accuracy in connecting to the federal net.

To promote the more general use of federal control stations, the U.S.C. and G.S. has transformed the geographic positions thereof to plane coordinates (X and Y). For each state a definite system of plane coordinates has been adopted, and all federal horizontal control therein is expressed on this plane. The purpose of transforming geographic positions to plane coordinates was to facilitate the use of federal surveys by making provision for a simpler method of treating the computations involved for the local surveys when they are connected to federal triangulation schemes. These plane coordinate systems were adopted principally as a procedure by which the surveyor could connect his traverse surveys and adjust his work to fit into the federal net, utilizing only those simple computational methods which are in general use. Professors Coddington and Marshall show how plane coordinates may also be used to adjust a local triangulation scheme.

After the arcs of triangulation are about 25 miles apart, the U.S.C. and G.S. has found it advisable to cover the entire remaining intermediate area with control, adjusting it simulataneously. In extending the area triangulation, the main scheme stations are about 8 miles apart. Supplemental stations are established at about 4-mile intervals along the main highways. Corrections necessary to fit properly the new work into the major networks surrounding it are then distributed over a relatively large number of observations and can be assimilated without causing too great a distortion in any one measurement or observation.

The authors mention urban control in Pittsburgh, Pa., and Columbus, Ohio. Other cities where geodetic control connected to the federal net has been accomplished are Rochester, N. Y.; Atlanta, Ga.; Boston, Mass.; Cleveland, Ohio; Hartford, Conn.; Minneapolis, Minn.; and Houston, Tex.; as well as many others. In June, 1947, the East Bay cities of the San Francisco (Calif.) area completed a comprehensive scheme of urban control rigidly connected to the federal net; and in December, 1947, the urban control for Cincinnati, Ohio, also rigidly connected to the federal net, was completed.

In the extension of the federal triangulation control, it is the policy of the U.S.C. and G.S. to establish base lines for length control in proximity to urban centers. These control the distance determinations in the federal net and will likewise aid in the control of the lengths for the urban triangulation whenever the cities establish a scheme of urban control. If such local control is available when the federal surveys are established in the locality, connections are made between the two schemes; and, if the bases and lengths of the urban control are of sufficient accuracy, the federal net is adjusted thereto.

The decade from 1937 to 1947 has marked a greatly augmented use of federal triangulation by engineers of state, municipal, and private organizations. It is confidently expected, that, as the ease and facility with which local survey connections may be made to the federal net become more generally known, this precise control will be used even more. Such papers as this are indicative that general interest is becoming increasingly manifest.

LANSING G. SIMMONS,[13] M. ASCE.—This subject has become of increasing interest in recent years, particularly to engineers and surveyors taking advantage of the rapidly expanding precise triangulation net of the United States. With the establishment of the state plane coordinate systems, the problem of adjusting triangulation directly on these plane grids, without reference to the geographic elements of position, length, and azimuth, is of concern to all involved in the establishment of high-grade horizontal control surveys. The authors have presented a solution which, although departing from correct theory in some instances, will produce excellent results for control schemes of local extent.

The writer cannot agree with the statement in the "Synopsis" to the effect that "Local triangulation nets, both urban and rural, can be precisely adjusted to large-scale geodetic triangulation with comparatively little effort * * *." Most readers will agree, particularly when it is realized that the actual solution of the nineteen simultaneous equations of the Logan County example does not appear in the paper. The importance and precision of the work should be considered before such a plan is adopted.

The new features brought out are the development and the use of condition equations, in terms of rectangular coordinates, necessary for the elimination of the position closure between stations previously established. This has always been a troublesome point in the adjustment of local work, and has usually been handled by simply neglecting the condition and by permitting inconsistencies to exist between new and old work.

Any triangulation net worthy of an adjustment, including the rigid conditions of position closures, is usually worthy of a completely rigid and simultaneous adjustment. The authors state that one of three possibilities for a preliminary adjustment would be to include all condition equations except those for latitude and departure closures. These would be introduced later in a final adjustment. It is believed that all conditions, including those of position, should be contained in one adjustment. This system not only gives better results in general, but can be accomplished more rapidly than any combination of successive adjustments that would yield comparable results.

Although the paper is principally confined to local triangulation nets, readers should be warned of the fact that, in strictness, the observed angles of a a triangulation scheme are not plane angles and should not be so used in working on a plane grid. The difference is infinitesimal for short lines on a local grid, but it becomes quite appreciable in carrying a precise scheme across a state with longer lines. These differences between observed (geodetic) angles and grid angles result from the fact that a straight line on the earth is not exactly a straight line on the projection, and vice versa. Their magnitude can easily approach 1 sec or 2 sec and, at times, several seconds of arc.

As an example, in carrying the computation over a considerable distance, if the necessary corrections are not applied to the observed angles to obtain grid angles, no account is made of the slowly changing scale of the projection. This difference between geodetic and grid angles can also be illustrated by the fact that a large geodetic triangle must necessarily close to 180° plus the

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spherical excess, whereas a triangle on the grid is closed to exactly 180° in an adjustment. Therefore, it follows that that the algebraic sum of the corrections to the three angles of a triangle must be equal to the spherical excess and negative in sign.

In conclusion, the authors of the paper have dealt with an interesting subject and have effected a nice solution. The foregoing cautions are aimed at those who propose to adapt the method to extended triangulation of high precision. It is believed that the most satisfactory solution in these cases is to compute the work geodetically, converting the position of each station to the grid individually.